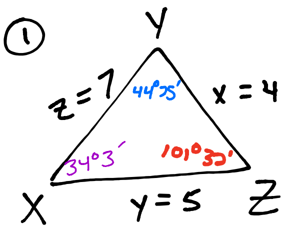
Solving Triangles

12.2 Practice – Law of Cosine & Sine

Determine the number of possible solutions. If a solution exists, solve the triangle. Round angle measures to the nearest minute and side measures to the nearest tenth.

#1) $x = 4$, $y = 5$, $z = 7$

①



One solution due to SSS

FIND LARGEST ANGLE 1st

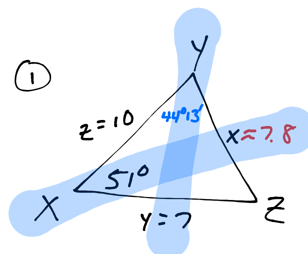
②
$z^2 = x^2 + y^2 - 2xy\cos Z$
$(7)^2 = (4)^2 + (5)^2 - 2(4)(5)\cos Z$
$49 = 16 + 25 - 40\cos Z$
$49 = 41 - 40\cos Z$
$8 = -40\cos Z$
$\frac{8}{-40} = \cos Z$
$\cos^{-1}\left(\frac{8}{-40}\right) = Z$
$101°32' \approx Z$

③
$y^2 = x^2 + z^2 - 2xz\cos Y$
$(5)^2 = (4)^2 + (7)^2 - 2(4)(7)\cos Y$
$25 = 16 + 49 - 56\cos Y$
$25 = 65 - 56\cos Y$
$-40 = -56\cos Y$
$\frac{40}{56} = \cos Y$
$\cos^{-1}\left(\frac{40}{56}\right) = Y$
$44°25' \approx Y$

④
$m\angle X + 101°32' + 44°25' = 180°$
$m\angle X + 145°57° = 180°$
$m\angle X = 34°3'$

#2) $z = 10$, $y = 7$, $m\angle X = 51°$

①



1 solution due to ASA

②
$x^2 = y^2 + z^2 - 2yz\cos X$
$x^2 = (7)^2 + (10)^2 - 2(7)(10)\cos 51°$
$x^2 = 49 + 100 - 140\cos 51°$
$x^2 = 149 - 140\cos 51°$
$x = \pm\sqrt{149 - 140\cos 51°}$
$x \approx 7.8$

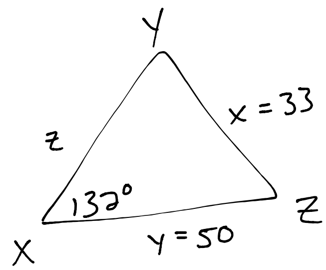
Next? Law of Sines OR Cosines.

③
$\frac{\sin 51°}{7.8} \approx \frac{\sin Y}{7}$
$\frac{7\sin 51°}{7.8} \approx \sin Y$
$\sin^{-1}\left(\frac{7\sin 51°}{7.8}\right) \approx Y$
$44°13' \approx Y$

④
$m\angle Z + 51° + 44°13' = 180°$
$m\angle Z + 95°13' = 180°$
$m\angle Z = 84°47'$

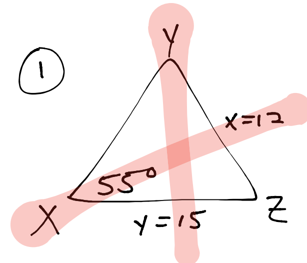
Solving Triangles
12.2 Practice – Law of Cosine & Sine

#3) $x = 33$, $y = 50$, $m\angle X = 132^\circ$



This Δ cannot exist. Since ∠X is the largest angle, side x should be the largest side but it isn't.

#4) $x = 12$, $y = 15$, $m\angle X = 55^\circ$



ASS!

2 $\dfrac{\sin 55^\circ}{12} = \dfrac{\sin Y}{15}$

$\dfrac{15\sin 55^\circ}{12} = \sin Y$

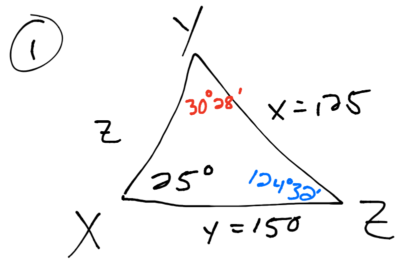
$\sin^{-1}\left(\dfrac{15\sin 55^\circ}{12}\right) = Y$

Error $= Y$

No Δ since ASS and ∠Y doesn't exist.

#5) $x = 125$, $y = 150$, $m\angle X = 25°$

①



ASS!

②
$$\frac{\sin 25°}{125} = \frac{\sin Y}{150}$$
$$\frac{150 \sin 25°}{125} = \sin Y$$
$$\sin^{-1}\left(\frac{150 \sin 25°}{125}\right) = Y$$
$$30°28' = Y$$

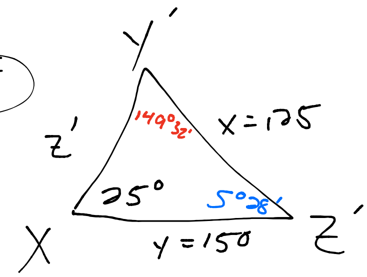
Since ASS and ∠Y is acute, there might be 2 solutions

③
$$25° + 30°28' + m\angle Z = 180°$$
$$55°28' + m\angle Z = 180°$$
$$m\angle Z = 124°32'$$

④
$$\frac{\sin 25°}{125} \approx \frac{\sin 124°32'}{z}$$
$$z \sin 25° \approx 125 \sin 124°32'$$
$$z \approx \frac{125 \sin 124°32'}{\sin 25°}$$
$$z \approx 243.7$$

Solution 2

⑤



⑥
$$m\angle Y + m\angle Y' = 180°$$
$$30°28' + m\angle Y' = 180°$$
$$m\angle Y' = 149°32'$$

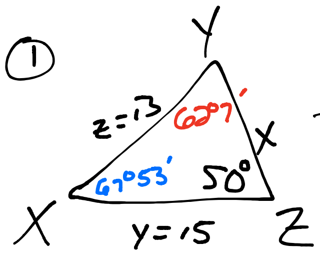
⑦
$$149°32' + 25° + m\angle Z' = 180°$$
$$174°32' + m\angle Z' = 180°$$
$$m\angle Z' = 5°28'$$

⑧
$$\frac{\sin 25°}{125} = \frac{\sin 5°28'}{z}$$
$$z \sin 25° = 125 \sin 5°28'$$
$$z = \frac{125 \sin 5°28'}{\sin 25°}$$
$$z \approx 28.2$$

#6) $z = 13$, $y = 15$, $m\angle Z = 50°$

Solution 1

1



ASS!

2

$$\frac{\sin 50°}{13} = \frac{\sin Y}{15}$$

$$\frac{15\sin 50°}{13} = \sin Y$$

$$\sin^{-1}\left(\frac{15\sin 50°}{13}\right) = Y$$

$$62°7' \approx Y$$

Maybe 2 solutions cuz ASS and ∠Y is acute

3

$$m\angle X + 62°7' + 50° = 180°$$

$$m\angle X + 112°7' = 180°$$

$$m\angle X = 67°53'$$

4

$$\frac{\sin 50°}{13} = \frac{\sin 67°53'}{x}$$

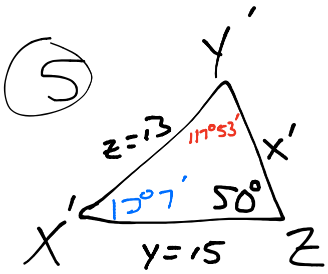
$$x\sin 50° = 13\sin 67°53'$$

$$x = \frac{13\sin 67°53'}{\sin 50°}$$

$$x \approx 15.7$$

Solution 2

5



6

$$m\angle Y + m\angle Y' = 180°$$

$$62°7' + m\angle Y' = 180°$$

$$m\angle Y' = 117°53'$$

7

$$m\angle X' + 50° + 117°53' = 180°$$

$$m\angle X' + 167°53' = 180°$$

$$m\angle X' = 12°7'$$

8

$$\frac{\sin 50°}{13} = \frac{\sin 12°7'}{x'}$$

$$x'\sin 50° = 13\sin 12°7'$$

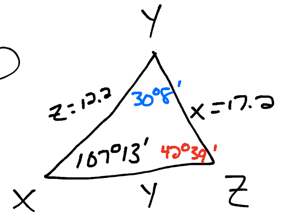
$$x' = \frac{13\sin 12°7'}{\sin 50°}$$

$$x' \approx 3.6$$

Solution 1

#7) $z = 12.2$, $x = 17.2$, $m\angle X = 107°13'$

1



ASS!

2

$$\frac{\sin 107°13'}{17.2} = \frac{\sin Z}{12.2}$$

$$\frac{12.2 \sin 107°13'}{17.2} = \sin Z$$

$$\sin^{-1}\left(\frac{12.2 \sin 107°13'}{17.2}\right) = Z$$

$$42°39' \approx Z$$

Maybe 2 solutions due to ASS and Z is acute

3

$$m\angle Y + 107°13' + 42°39' = 180°$$

$$m\angle Y + 149°52' = 180°$$

$$m\angle Y = 30°8'$$

4

$$\frac{\sin 107°13'}{17.2} = \frac{\sin 30°8'}{y}$$

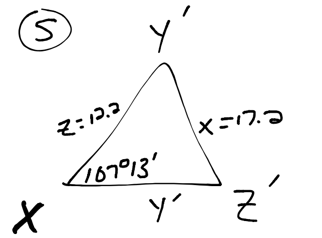
$$y \sin 107°13' = 17.2 \sin 30°8'$$

$$y = \frac{17.2 \sin 30°8'}{\sin 107°13'}$$

$$y \approx 9.0$$

Solution 2

5



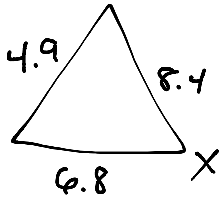
$$m\angle Z + m\angle Z' = 180°$$

$$42°39' + m\angle Z' = 180$$

$$m\angle Z' = 137°21'$$

Can't have 2 obtuse angles. So 1 solution

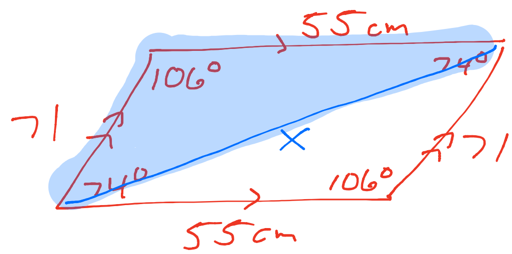
#8) The sides of a triangle measure 6.8cm, 8.4cm, and 4.9cm. Find the measure of the smallest angle to the nearest minute.



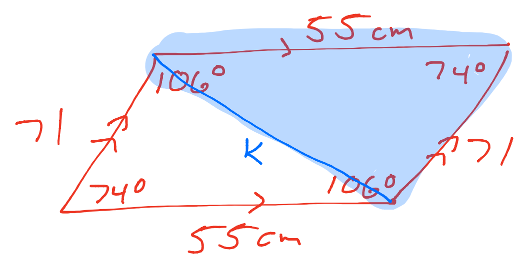
$$x^2 = y^2 + z^2 - 2yz\cos X$$
$$(4.9)^2 = (6.8)^2 + (8.4)^2 - 2(6.8)(8.4)\cos X$$
$$24.01 = 46.24 + 70.56 - 114.24\cos X$$
$$24.01 = 116.8 - 114.24\cos X$$
$$-92.79 = -114.24\cos X$$
$$\frac{92.79}{114.24} = \cos X$$
$$\cos^{-1}\left(\frac{92.79}{114.24}\right) = X$$
$$35°41' \approx X$$

The smallest angle is about 35°41'

#9) A parallelogram has sides of 55cm and 71cm. Find the length of each diagonal to the nearest tenth if the largest angles measures 106º.

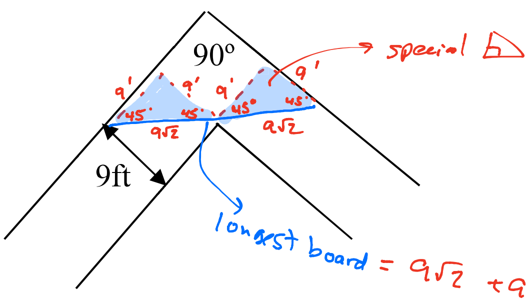


$$x^2 = y^2 + z^2 - 2yz\cos X$$
$$x^2 = (55)^2 + (71)^2 - 2(55)(71)\cos 106°$$
$$x^2 = 3025 + 5041 - 7810\cos 106°$$
$$x^2 = 8066 - 7810\cos 106°$$
$$x = \pm\sqrt{8066 - 7810\cos 106°}$$
$$x \approx 101.1 \text{ cm}$$



$$K^2 = y^2 + z^2 - 2yz\cos K$$
$$K^2 = (55)^2 + (71)^2 - 2(55)(71)\cos 74°$$
$$K^2 = 3025 + 5041 - 7810\cos 74°$$
$$K^2 = 8066 - 7810\cos 74°$$
$$K = \pm\sqrt{8066 - 7810\cos 74°}$$
$$K \approx 76.9 \text{ cm}$$

#10) Two carpenters are trying to carry a board down a hallway and around a corner in the Lawson Building. The figure is an overhead view of the hallway. What is the length of the longest board that can be carried parallel to the floor through the hallway, to the nearest tenth foot? (Hint you may want to make a scale drawing and experiment with little strips of paper. Lqtm.)

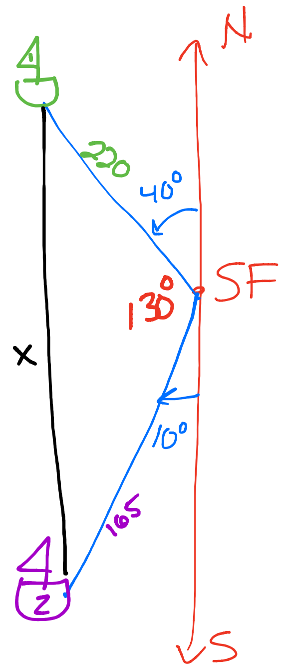


longest board $= 9\sqrt{2} + 9\sqrt{2}$

$= 18\sqrt{2}$ feet

$= 25.5$ feet

#11) Two ships leave San Fran at the same time. One travels 40° west of north at a speed of 20 knots. The other travels 10° west of south at a speed of 15 knots. How far apart are the ships after 11 hours?



$D_1 = r_1 t_1$

$D_1 = (20)(11)$

$D_1 = 220$

$D_2 = r_2 t_2$

$D_2 = (15)(11)$

$D_2 = 165$

$x^2 = y^2 + z^2 - 2yz\cos X$

$x^2 = (220)^2 + (165)^2 - 2(220)(165)\cos 130°$

$x^2 = 48400 + 27225 - 72600\cos 130°$

$x^2 = 75625 - 72600\cos 130°$

$x = \pm\sqrt{75{,}625 - 72{,}600\cos 130°}$

$x \approx 349.7$

The ships are 349.7 nautical miles apart

#1) $m\angle X = 34°3'$, $m\angle Y = 44°25'$, $m\angle Z = 101°32'$

#2) $x = 7.8$, $m\angle Y = 44°13'$, $m\angle Z = 84°47'$

#3) None

#4) None

#5) TWO:

$m\angle Z = 124°32'$, $m\angle Y = 30°28'$, $z = 243.7$;
$m\angle Z' = 5°28'$, $m\angle Y' = 149°32'$, $z' = 28.2$

#6) TWO: $x = 15.7$, $m\angle X = 67°53'$, $m\angle Y = 62°7'$; $x' = 3.6$, $m\angle X' = 12°6'$, $m\angle Y' = 117°54'$

#7) ONE: $y = 9.0$, $m\angle Y = 30°8'$, $m\angle Z = 42°39'$

#8) $m\angle$angle = 35º41'

#9) 101.1cm, 76.9cm

#10) 25.5ft

#11) 349.7 nautical miles