Trig Identities & Equations
Review Unit 11

Reciprocal Identities:

1. $\sin\theta = \frac{1}{\csc\theta}$
2. $\csc\theta = \frac{1}{\sin\theta}$
3. $\cos\theta = \frac{1}{\sec\theta}$
4. $\sec\theta = \frac{1}{\cos\theta}$
5. $\tan\theta = \frac{1}{\cot\theta}$
6. $\cot\theta = \frac{1}{\tan\theta}$

Quotient Identities:

1. $\tan\theta = \frac{\sin\theta}{\cos\theta}$
2. $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean Identities:

1. $\sin^2\theta + \cos^2\theta = 1$
2. $1 + \tan^2\theta = \sec^2\theta$
3. $1 + \cot^2\theta = \csc^2\theta$

Sum and Difference Identities

1. $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
2. $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
3. $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
4. $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
5. $\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$
6. $\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

Double Angle Identities

1. $\sin 2\theta = 2\sin\theta\cos\theta$
2. $\cos 2\theta = \cos^2\theta - \sin^2\theta$
3. $\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$

Half Angle Identities

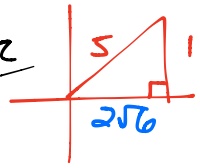
1. $\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$
2. $\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$
3. $\tan\frac{\alpha}{2} = \frac{\sin\alpha}{1+\cos\alpha}$, $\cos\alpha \neq -1$

Solve for values of θ between 0° and 90°.

#1) If csc θ = 5, find sec θ.

Step 1 $\csc\theta = \frac{5}{1} = \frac{r}{y}$

Step 2



Step 3 $x^2 + y^2 = r^2$

$x^2 + (1)^2 = (5)^2$

$x^2 + 1 = 25$

$x^2 = 24$

$x = \pm 2\sqrt{6}$

$x = 2\sqrt{6}$

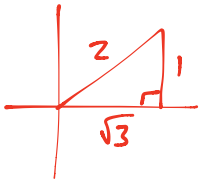
Step 4 $\sec\theta = \frac{r}{x}$

$\sec\theta = \frac{5}{2\sqrt{6}}$

$= \frac{5\sqrt{6}}{2 \cdot 6}$

$\sec\theta = \frac{5\sqrt{6}}{12}$

#2) If $\sin\theta = \frac{1}{2}$ ($y = 1$, $r = 2$), find cos θ.



30°-60°-90°

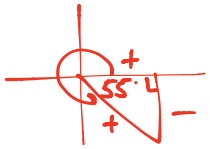
$\cos\theta = \frac{\sqrt{3}}{2}$

Express each value as a function of an angle in Quadrant I.

#3) sin 665°

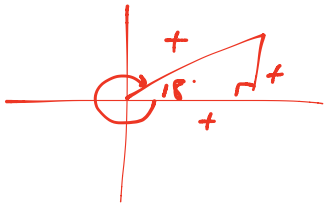
coterminal : 665 - 360°

: 305°



Answer: $-\sin 55°$

#4) tan –342°



Answer: $+\tan 18°$

Simplify.

#5) $\frac{\tan x \csc x}{\sec x}$ $= \frac{\tan x \cos x}{\sin x}$

$= \frac{\frac{\sin x}{\cos x} \cos x}{\sin x}$

$= \frac{\sin x}{\sin x}$

$= 1$

#6) $\tan A \cos^2 A$

$= \frac{\sin A}{\cos A} \cos^2 A$

$= \sin A \cos A$

Write an expression for each of the following in terms of the given function.

#7) sec θ in terms of cot θ

$1 + \tan^2\theta = \sec^2\theta$

$\pm\sqrt{1+\tan^2\theta} = \sec\theta$ ✓

$\pm\sqrt{1+\frac{1}{\cot^2\theta}} = \sec\theta$

$\pm\sqrt{\frac{\cot^2\theta}{\cot^2\theta}+\frac{1}{\cot^2\theta}} = \sec\theta$

$\pm\sqrt{\frac{\cot^2\theta+1}{\cot^2\theta}} = \sec\theta$

$\pm\frac{\sqrt{\cot^2\theta+1}}{\cot\theta} = \sec\theta$

#8) csc θ in terms of cos θ

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\sin\theta = \pm\sqrt{1-\cos^2\theta}$

$\frac{1}{\csc\theta} = \pm\sqrt{1-\cos^2\theta}$

$1 = \csc\theta\left(\pm\sqrt{1-\cos^2\theta}\right)$

$\frac{1}{\pm\sqrt{1-\cos^2\theta}} = \csc\theta$

$\frac{\pm\sqrt{1-\cos^2\theta}}{1-\cos^2\theta} = \csc\theta$

Verify that each of the following is an identity.

#9) $\frac{1}{\sec^2\phi} + \frac{1}{\csc^2\phi} = 1$

$\cos^2\theta + \sin^2\theta = 1$

#10) $\frac{\tan\alpha\cos\alpha}{\sin\alpha} = 1$

$\frac{\frac{\sin\alpha}{\cos\alpha}\cos\alpha}{\sin\alpha} =$

$\frac{\sin\alpha}{\sin\alpha} =$

Find a numerical value of one trig function of each x.

#11) tan x cos x = ½

$\frac{\sin x}{\cos x}\cos x = \frac{1}{2}$

$\sin x = \frac{1}{2}$

#12) $\frac{\tan x}{\sin x} = \sqrt{2}$

$\frac{\sin x}{\cos x} \cdot \frac{1}{\sin x} =$

$\frac{1}{\cos x} =$

$\sec x = \sqrt{2}$

Use the sum and difference identities to find the exact value of each function.

#13) cos 255°

$= \cos(45° + 210°)$

$= \cos 45° \cos 210° - \sin 45° \sin 210°$

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{-\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{-1}{2}\right)$

$= \frac{-\sqrt{6}}{4} - \frac{-\sqrt{2}}{4}$

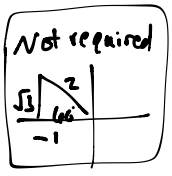
$= \frac{-\sqrt{6}+\sqrt{2}}{4}$

$\cos 255° = \frac{\sqrt{2}-\sqrt{6}}{4}$

#14) tan 165° $= \tan(45° + 120°)$

$= \frac{\tan 45° + \tan 120°}{1 - \tan 45° \tan 120°}$

Not required



$= \frac{1 + (-\sqrt{3})}{1 - (1)(-\sqrt{3})}$

$= \frac{(1-\sqrt{3})}{(1+\sqrt{3})}\frac{(1-\sqrt{3})}{(1-\sqrt{3})}$

$= \frac{1 - 2\sqrt{3} + 3}{1 - 3}$

$= \frac{4 - 2\sqrt{3}}{-2}$

$= \frac{-2(-2+\sqrt{3})}{-2}$

$\tan 165° = \sqrt{3} - 2$

If α and β are the measures of two first quadrant angles, find the exact value of each function.

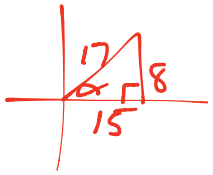
#15) If $\sin\alpha = \frac{8}{17}$ and $\tan\beta = \frac{7}{24}$ find $\cos(\alpha - \beta)$.

$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$= \left(\frac{15}{17}\right)\left(\frac{24}{25}\right) + \left(\frac{8}{17}\right)\left(\frac{7}{25}\right)$

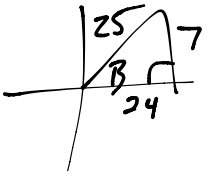
$= \frac{360}{425} + \frac{56}{425}$

$\cos(\alpha - \beta) = \frac{416}{425}$



Pythag. Triple!

$\cos\alpha = \frac{15}{17}$



$\sin\beta = \frac{7}{25}$

$\cos\beta = \frac{24}{25}$

#16) If $\csc\alpha = \frac{13}{5}$ and $\tan\beta = \frac{3}{4}$, find $\tan(\alpha + \beta)$.

$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

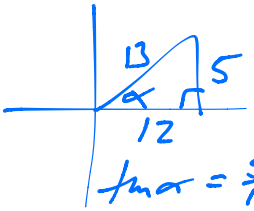
$= \frac{\left(\frac{5}{12}\right) + \left(\frac{3}{4}\right)}{1 - \left(\frac{5}{12}\right)\left(\frac{3}{4}\right)}$

$= \frac{\frac{5}{12} + \frac{9}{12}}{\frac{16}{16} - \frac{5}{16}}$

$= \frac{\frac{14}{12}}{\frac{11}{16}}$

$= \frac{7}{6} \cdot \frac{16}{11}$

$\tan(\alpha+\beta) = \frac{56}{33}$



Pythag. Triple!

$\tan\alpha = \frac{5}{12}$

Trig Identities & Equations
Review Unit 11

Verify that each of the following is an identity.

#17) $\tan(270° - x) = \cot x$

$$\frac{\tan 270° - \tan x}{1 + \tan 270° \tan x} =$$

$$\frac{\text{und} - \tan x}{1 + \text{und} \cdot \tan x} =$$

Reboot $\frac{\sin(270° - x)}{\cos(270° - x)} =$

$$\frac{\sin 270° \cos x - \cos 270° \sin x}{\cos 270° \cos x + \sin 270° \sin x} =$$

$$\frac{(-1)\cos x - (0)\sin x}{(0)\cos x + (-1)\sin x} =$$

$$\frac{+\cos x}{+\sin x} =$$

$\cot x = \cot x$

#18) $\cos(360° - \gamma) = \cos \gamma$

$\cos 360° \cos\gamma + \sin 360° \sin\gamma =$

$(1)\cos\gamma + (0)\sin\gamma =$

$\cos\gamma = \cos\gamma$

If $\sin B = \frac{3}{5}$ and B is in the first quadrant, find each value.

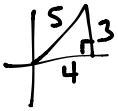
#19) $\cos 2B$

$= \cos^2 B - \sin^2 B$

$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$= \frac{16}{25} - \frac{9}{25}$

$\cos 2B = \frac{7}{25}$



QUADI so +

#20) $\tan\frac{B}{2} = \pm\frac{\sin B}{1 + \cos B}$

$$= \frac{\left(\frac{3}{5}\right)}{\left(\frac{5}{5}\right) + \left(\frac{4}{5}\right)}$$

$$= \frac{\frac{3}{5}}{\frac{9}{5}}$$

$$= \frac{3}{9}$$

$\tan\frac{B}{2} = \frac{1}{3}$

QUADI so +

#21) $\sin\frac{B}{2} = \pm\sqrt{\frac{1-\cos B}{2}}$

QUAD I so +

$= +\sqrt{\frac{\frac{3}{5}-\left(\frac{4}{5}\right)}{2}}$

$= \sqrt{\frac{\frac{1}{5}}{2}}$

$= \sqrt{\frac{1}{10}}$

$= \frac{\sqrt{1}}{\sqrt{10}}$

$\sin\frac{B}{2} = \frac{\sqrt{10}}{10}$

#23) $\cos\frac{\frac{13\pi}{12}\cdot 2}{1\cdot 2} = \cos\frac{\frac{13\pi}{6}}{2}$

QUAD III so −

$= \pm\sqrt{\frac{1+\cos\frac{13\pi}{6}}{2}}$

$= -\sqrt{\frac{\frac{2}{2}+\frac{\sqrt{3}}{2}}{2}}$

$= -\sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}}$

$= -\sqrt{\frac{2+\sqrt{3}}{4}}$

$\cos\frac{13\pi}{12} = -\frac{\sqrt{2+\sqrt{3}}}{2}$

Use a half-angle identity to find each value.

#22) $\tan\frac{195^\circ\cdot 2}{1\cdot 2} = \tan\frac{390^\circ}{2}$

QUAD III so +

$= \pm\frac{\sin 390^\circ}{1+\cos 390^\circ}$

$= \frac{\frac{1}{2}}{\frac{2}{2}+\frac{\sqrt{3}}{2}}$

$= \frac{\frac{1}{2}\cdot 2}{\frac{2+\sqrt{3}}{2}\cdot 2}$

$= \frac{1}{2+\sqrt{3}}$

$= \frac{2-\sqrt{3}}{4-3}$

$= \frac{2-\sqrt{3}}{1}$

$\tan 195^\circ = 2-\sqrt{3}$

this is the same as

$\sqrt{7-4\sqrt{3}}$

Verify that each of the following is an identity.

#24) $\sin 2x = 2\cot x \sin^2 x$

$2\sin x\cos x = 2\cdot\frac{\cos x}{\sin x}\cdot\sin^2 x$

$= 2\cos x\sin x$

$2\sin x\cos x = 2\sin x\cos x$

#25) $\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$

$\sin^2\theta = \frac{1}{2}\left[1 - (\cos^2\theta - \sin^2\theta)\right]$

$= \frac{1}{2}\left[1 - \cos^2\theta + \sin^2\theta\right]$

$= \frac{1}{2}\left[\sin^2\theta + \sin^2\theta\right]$

$= \frac{1}{2}\left[2\sin^2\theta\right]$

$\sin^2\theta = \sin^2\theta$

#1) $\frac{5\sqrt{6}}{12}$

#2) $\frac{\sqrt{3}}{2}$

#3) $-\sin 55°$

#4) $\tan 18°$

#5) 1

#6) $\sin A \cos A$

#7) $\pm\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$

#8) $\pm\frac{\sqrt{1-\cos^2\theta}}{1-\cos^2\theta}$

#11) sample answer: $\sin x = \frac{1}{2}$

#12) sample answer: $\sec x = \sqrt{2}$

#13) $\frac{\sqrt{2}-\sqrt{6}}{4}$

#14) $\sqrt{3}-2$

#15) $\frac{416}{425}$

#16) $\frac{56}{33}$

#17) answers vary

#18) answers vary

#19) $\frac{7}{25}$

#20) $\frac{1}{3}$

#21) $\frac{\sqrt{10}}{10}$

#22) $\sqrt{7-4\sqrt{3}}$

#23) $-\frac{\sqrt{2+\sqrt{3}}}{2}$

#24) answers vary

#25) answers vary