$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$ $\qquad$ $(a+b)^n = \sum_{k=0}^{n}\binom{n}{k}a^{n-k}b^k$

$S_n = \frac{n(a_1+a_n)}{2}$

Unit 14 Review

Find the next three terms in each sequence. Then, tell if the sequence converges or diverges and write the explicit rule.

1) $5,\ 1,\ \frac{1}{5},\ \frac{1}{25},\ \frac{1}{125},\ \ldots$ $\frac{1}{625},\ \frac{1}{3125},\ \frac{1}{15625}$

Converges to 0

Explicit Rule: $5\cdot\left(\frac{1}{5}\right)^{n-1}$ or $\left(\frac{1}{5}\right)^{n-2}$

2) $-\frac{4}{3},\ -\frac{2}{3},\ 0,\ \frac{2}{3},\ \frac{4}{3},\ \ldots$ $\frac{6}{3},\ \frac{8}{3},\ \frac{10}{3}$

Diverges

Explicit Rule: $-\frac{4}{3}+\frac{2}{3}(n-1)$

Find the first four terms in each sequence.

3) $a_n = 2^n - 2$

$a_1 = 2^1 - 2 = 0$
$a_2 = 2^2 - 2 = 2$
$a_3 = 2^3 - 2 = 6$
$a_4 = 2^4 - 2 = 14$

4) $a_n = 62 - 30n$

$a_1 = 62 - 30(1) = 62 - 30 = 32$
$a_2 = 62 - 30(2) = 62 - 60 = 2$
$a_3 = 62 - 30(3) = 62 - 90 = -28$
$a_4 = 62 - 30(4) = 62 - 120 = -58$

5) $a_n = a_{n-1} + 2$
$a_1 = -24$

$a_2 = (-24) + 2 = -22$
$a_3 = (-22) + 2 = -20$
$a_4 = (-20) + 2 = -18$

6) $a_n = a_{n-1} + \frac{1}{3}$
$a_1 = -6$

$a_2 = (-6) + \frac{1}{3} = -\frac{17}{3}$
$a_3 = \left(-\frac{17}{3}\right) + \frac{1}{3} = -\frac{16}{3}$
$a_4 = \left(-\frac{16}{3}\right) + \frac{1}{3} = -\frac{15}{3} = -5$

Write the explicit formula for each sequence.

7) $-3,\ -6,\ -12,\ -24,\ -48,\ \ldots$ Geometric Sequence: $r = 2$

$a_n = -3\left(2^{n-1}\right)$

8) $-4,\ -14,\ -24,\ -34,\ -44,\ \ldots$

Arithmetic Sequence $d = -10$

$a_n = -4 + (n-1)(-10)$

Write the recursive formula for each sequence.

9) $1,\ 4,\ 16,\ 64,\ 256,\ \ldots$

$a_n = 4a_{n-1}$
$a_1 = 1$

10) $-13,\ 17,\ 47,\ 77,\ 107,\ \ldots$

$a_n = 30 + a_{n-1}$
$a_1 = -13$

Evaluate each series.

11) $\sum_{n=0}^{5}\left(20 - n^2\right)$

$= (20-0^2)+(20-1^2)+(20-2^2)+(20-3^2)+(20-4^2)+(20-5^2)$
$= (20-0)+(20-1)+(20-4)+(20-9)+(20-16)+(20-25)$
$= 20 + 19 + 16 + 11 + 4 - 5$
$= 65$

12) $\sum_{m=0}^{4} 3m$ $= 3(0)+3(1)+3(2)+3(3)+3(4)$
$= 0 + 3 + 6 + 9 + 12$
$= 30$

Rewrite each series using sigma notation.

13) $3 + 9 + 27 + 81 + 243$

$\sum_{n=1}^{5} 3^n$

14) $601 + 602 + 603 + 604 + 605 + 606$

$\sum_{n=601}^{606} n$

For each sequence, state if it is arithmetic, geometric, or neither. If it is arithmetic, tell the common difference. If it is geometric, tell the common ratio. Then, find the sum of the first 50 terms.

15) $\cancel{1}\,\frac{2}{2},\ \frac{3}{2},\ \cancel{2}\,\frac{4}{2},\ \frac{5}{2},\ \cancel{3}\,\frac{6}{2},\ \ldots$

Arithmetic, $d = \frac{1}{2}$

$S_n = \frac{n(a_1 + a_n)}{2}$

$S_{50} = \frac{50(1 + 25.5)}{2}$

$S_{50} = 25(26.5)$

$S_{50} = 662.5$

$a_n = a_1 + (n-1)d$

$a_{50} = 1 + 49\left(\frac{1}{2}\right)$

$= \frac{2}{2} + \frac{49}{2}$

$= \frac{51}{2}$

$a_{50} = 25.5$

16) $-108,\ 18,\ -3,\ \frac{1}{2},\ -\frac{1}{12},\ \ldots$

Geometric, $r = -\frac{1}{6}$

$S_n = a_1\left(\frac{1 - r^n}{1 - r}\right)$

$S_{50} = -108\left(\frac{1 - \left(-\frac{1}{6}\right)^{50}}{1 - \left(-\frac{1}{6}\right)}\right)$

$S_{50} = -92.5714$ → Frac

$S_{50} = \frac{-648}{7}$

Evaluate each expression.

17) ${}_{10}C_5 = \frac{10!}{5!5!} = \frac{\cancel{10}^{2} \cdot 9 \cdot \cancel{8}^{2} \cdot 7 \cdot \cancel{6} \cdot \cancel{5!}}{\cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1 \cdot \cancel{5!}} = 252$

18) ${}_{12}C_7 = \frac{12!}{7!5!} = \frac{\cancel{12} \cdot 11 \cdot \cancel{10} \cdot 9 \cdot 8 \cdot \cancel{7!}}{\cancel{7!}\ \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1} = 792$

Find each term described.

19) 2nd term in expansion of $(x + 3)^3$

1
1 1
1 2 1
1 3 3 1

2nd term: $3(x)^2(3)^1$

: $9x^2$

20) 4th term in expansion of $(3u - 1)^4$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

4th term: $4(3u)^1(-1)^3$

: $4 \cdot 3u(-1)$

: $-12u$

Expand completely.

21) $(2y - x)^4$

$= 1(2y)^4(-x)^0 + 4(2y)^3(-x)^1 + 6(2y)^2(-x)^2 + 4(2y)^1(-x)^3 + 1(2y)^0(-x)^4$

$= 1 \cdot 16y^4 \cdot 1 + 4 \cdot 8y^3(-x) + 6 \cdot 4y^2 \cdot x^2 + 4 \cdot 2y(-x^3) + 1 \cdot 1 \cdot x^4$

$= 16y^4 - 32xy^3 + 24x^2y^2 - 8x^3y + x^4$

22) $(y - 3x)^3$

$= 1(y)^3(-3x)^0 + 3(y)^2(-3x)^1 + 3(y)^1(-3x)^2 + 1(y)^0(-3x)^3$

$= 1 \cdot y^3 \cdot 1 + 3y^2(-3x) + 3y \cdot 9x^2 + 1 \cdot 1(-27x^3)$

$= y^3 - 9xy^2 + 27x^2y - 27x^3$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1