

## 2.2 Application and Extension

**Relevant domain** for a model is a domain that fits the situation. For each of the following models (27-30) give the relevant domain using inequality notation and explain your reasoning in a full sentence. Some problems will have more than one correct answer, so do your best to explain.

27. The volume of a sphere depends on its radius and is modeled by  $V(r) = \frac{4}{3}\pi r^3$ .

$0 < r \leq 3959 \text{ mi}$

For a sphere to exist, it must have a positive radius. The largest sphere on planet Earth is planet Earth with radius 3959 mi.

28. A person's height (feet) depends on their age (years) and is modeled by  $H(a) = \frac{1}{4}a + 1$

$0 < a \leq 122$

A human's age must be positive and the oldest human to live in modern times was 122 years old.

29. The distance you travel while hiking is a function of how long you hike at 3 miles per hour. This is modeled by  $d(t) = 3t$  where  $t$  is measured in hours.

$0 < t < 16$

Unless time traveling, time must be positive. Most people would hike for less than 16 hours per day.

30. To change Celsius to Fahrenheit, use the formula  $F(C) = \frac{9}{5}C + 32$ . You are concerned only with temperatures from **freezing to boiling**.

$0 \leq C \leq 100^\circ$

Water freezes at  $0^\circ\text{C}$  and boils at  $100^\circ\text{C}$ .

31. Which function is only decreasing?

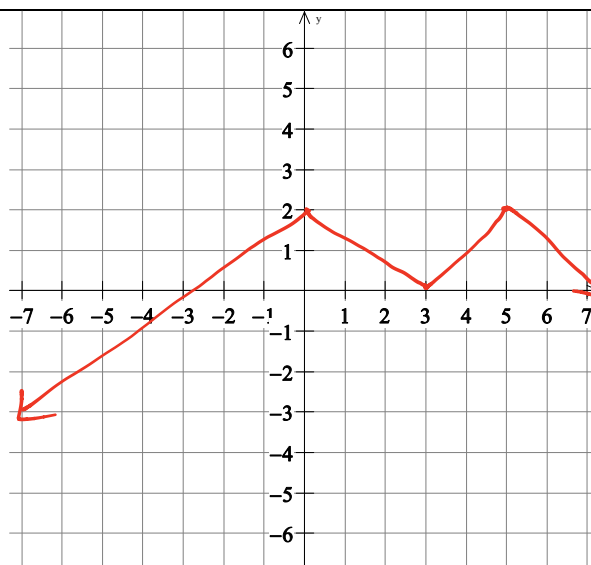
- (A) Outdoor temperature as a function of time.
- (B) The Dow Jones Industrial Average as a function of time.
- (C) Air pressure in the Earth's atmosphere as a function of altitude.
- (D) World population since 1900 as a function of time.
- (E) Water pressure in the ocean as a function of depth.

32. If a graph's *average* slope between two points is positive, then is the graph increasing or decreasing?

Increasing

33. Sketch (freehand) a graph of a function  $f$  with domain all real numbers that satisfies all of the following conditions:

- a. There are no breaks in the graph (it is continuous).
- b.  $f$  is increasing on  $(-\infty, 0)$  and on  $(3, 5)$
- c.  $f$  is decreasing on  $(0, 3)$  and on  $(5, \infty)$
- d.  $f(0) = f(5) = 2$
- e.  $f(3) = 0$



34. Mr. Brust's flip-flops are thrown into the air by a catapult. Their height (measured in feet) is modeled by the equation  $h(t) = -16t^2 + 85t$ , where  $t$  is seconds.

I. What is the relevant domain of this model (use a graphing calculator)? What does it represent?

Domain =  $[0, 5.3125]$

The domain represents the time the flip-flops were in the air.  
It also shows when the flip-flops were on the ground.

II. What is the relevant range of this model (Use a graphing calculator)? What does it represent?

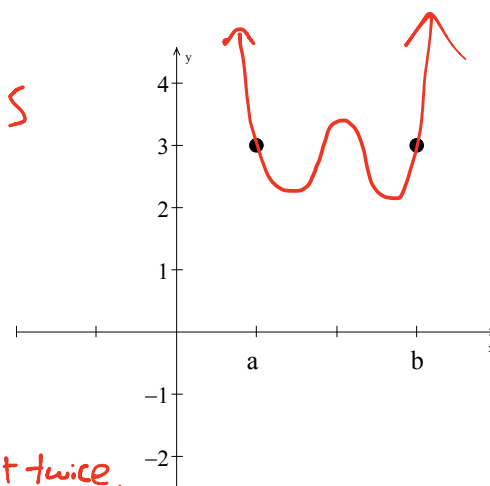
RANGE =  $[0, 112.891]$

The range represents all the possible heights of the flip-flops.

35. Let  $f$  be a polynomial function with degree greater than 2 (cubic, quartic, etc). If  $a \neq b$  and  $f(a) = f(b) = 3$ , which of the following must be true for at least one value of  $x$  between  $a$  and  $b$ . (One, both, or neither could be true.) Explain your reasoning in full sentences.

I.  $f(x) = 0$  at least once. **NO**

II.  $f(x)$  has at least one maximum or minimum. **YES**



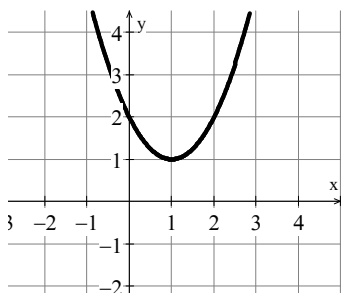
**Hint:** Use the graph to help you draw a picture that proves or disproves each statement.

I. is false because the graph could be a quartic, opening up, with vertex above the x-axis. In this case,  $y$  never equals zero.

II. Any polynomial function has a minimum of one minimum or maximum if it has the same height twice.

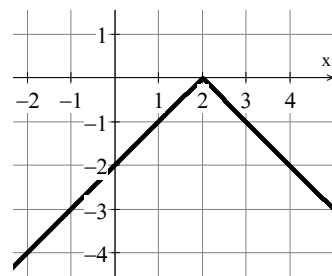
**Skillz Review:** Write the function of each graph using  $f(x) = \sqrt{x}$ ,  $f(x) = x^3$ ,  $f(x) = |x|$ , or  $f(x) = x^2$ .

1)  $f(x) =$



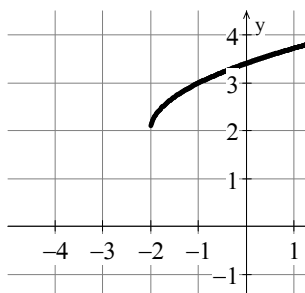
$y = (x-1)^2 + 1$

2)  $f(x) =$



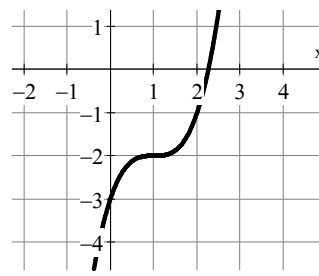
$y = -|x-2|$

3)  $f(x) =$



$y = \sqrt{x+2} + 2$

4)  $f(x) =$



$y = (x-1)^3 - 2$