3.2 Application and Extension

For 1-2, the extrema are listed for a function f along with the restricted domain. Find the absolute maximum value and absolute minimum value on the interval. DON'T USE THE GRAPH!

1. $f(x) = 0.7x^3 - 3x^2 + x;\ -1 \le x \le 4$

Evaluate w/calc

Extrema at: $f(-1) = -4.7$

$x = 0.178$ $f(4) = 0.8$ ABS MAX

$x = 2.679$

$f(0.178) = 0.087$

$f(2.679) = -5.393$ ABS MIN

2. $f(x) = -21x^4 - 9x^3 + 50x^2 + 13x;$ Evaluate w/calc

$-2 \le x \le 1.2$

$f(-2) = -90$ ABS MIN

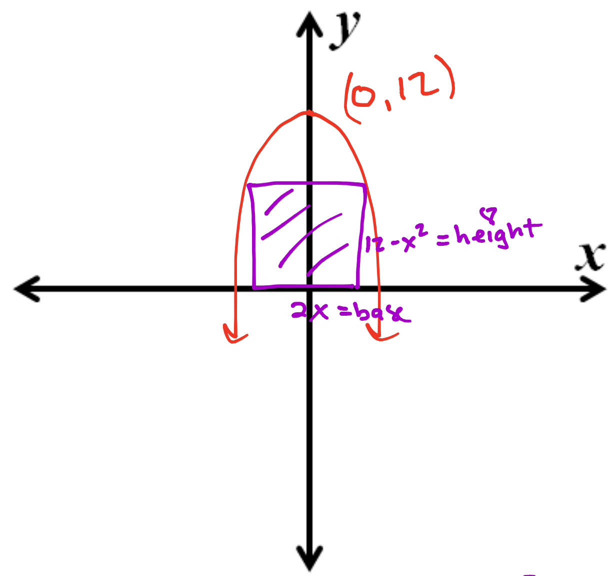
Extrema at: $f(1.2) = 28.502$

$x = -1.204$ $f(-1.204) = 28.408$

$x = -0.127$ $f(-0.127) = -0.832$

$x = 1.01$ $f(1.01) = 33.01$ ABS MAX

3. A rectangle has its base on the x-axis and its two upper corners on the parabola $y = 12 - x^2$.



a. Draw this scenario on the coordinate plane to the right, and draw a possible rectangle.

b. Label the base and height of your rectangle in terms of x.

c. Find the function $A(x)$ that represents the area of the rectangle. $A(x) = bh$

$= 2x(12 - x^2)$

$A(x) = 24x - 2x^3$

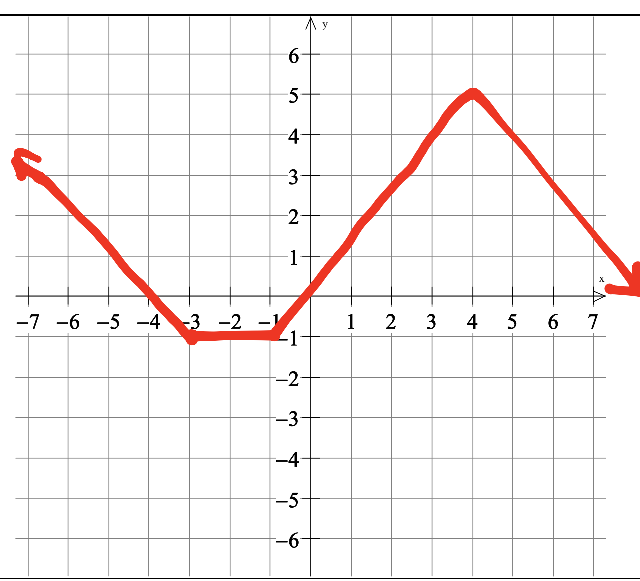
d. What is the largest possible area of this rectangle? (*Hint: Use a calculator to graph and find the maximum!*) FIND MAX of $A(x) = 24x - 2x^3$

$A(x) = 32$ un²

e. At what x-value should the rectangle be drawn for the largest area?

$x = 2$ units

4. Sketch (freehand) a graph of a function g with domain all real numbers that satisfies all of the following conditions:



✓ a. There are no breaks in the graph (it is continuous).

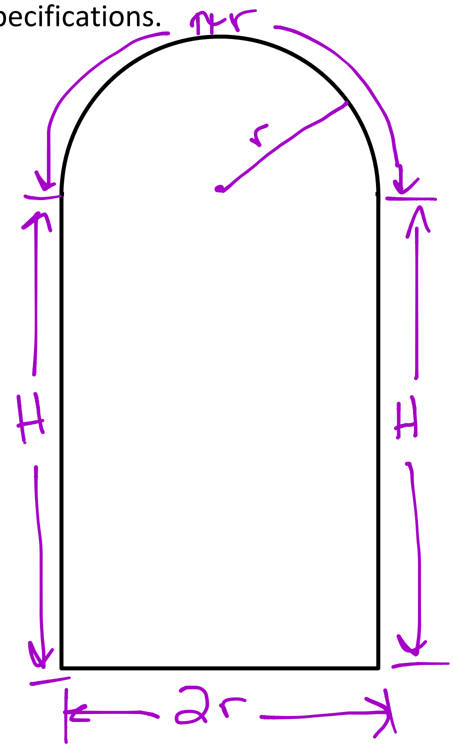
✓ b. g is decreasing on $(-\infty, -3)$ and on $(4, \infty)$

✓ c. g is increasing on $(-1, 4)$

✓ d. $g(4) > g(-7)$

✓ e. $g(x) < 0$ on $(-4, 0)$

5. Mr. Sullivan has hired you to design a window in front of his house. His specifications are that it is to be a rectangular shape with a semi-circle on top (see figure) and the perimeter of the window is 288 inches. He wants you to create a window with the largest possible area that fits those specifications.



a. Using r as the radius of the semi-circle, label the top edge of the semi-circle in terms of r. (*Hint: What is the circumference of a circle?*)

If $C = 2\pi r$, then $\frac{1}{2}C = \pi r$

b. Label the bottom of the window in terms of r.

c. Label the height of the rectangular portion as H.

d. Find H in terms of r.

$$\text{Perimeter} = H + H + 2r + \pi r$$
$$288 = 2H + 2r + \pi r$$
$$288 - 2r - \pi r = 2H$$
$$144 - r - \frac{\pi r}{2} = H$$

e. Label the area of the semi-circle $a_1(r)$. Find an equation for $a_1(r)$.

$$a_1(r)_{\circ} = \frac{1}{2}A_{\circ}$$
$$a_1(r)_{\circ} = \frac{1}{2}\pi r^2$$

f. Label the area of the rectangle $a_2(r)$. Find an equation for $a_2(r)$.

$$a_2(r)_{\square} = b \cdot h$$
$$= 2r \cdot H$$
$$= 2r\left(144 - r - \frac{\pi r}{2}\right)$$
$$a_2(r)_{\square} = 288r - 2r^2 - \pi r^2$$

g. Find $A(r)$, the total area of the window.

$$A(r) = a_1(r) + a_2(r)$$
$$= \left(\frac{1}{2}\pi r^2\right) + \left(288r - 2r^2 - \pi r^2\right)$$
$$A(r) = -2r^2 + 288r - \frac{1}{2}\pi r^2$$

h. What is the largest area of the window? (3 decimal places and use correct units)

MAX $A(r) \approx 5807.108 \text{ in}^2$

i. What is the width of the bottom of the window to create this large area? (3 decimal places and use correct units)

$r = 40.327138$

$2r = 80.654$ inches