3.2 Application and Extension

For 1-2, the extrema are listed for a function $f$ along with the restricted domain. Find the absolute maximum value and absolute minimum value on the interval. DON'T USE THE GRAPH!

1. $f(x) = 0.7x^3 - 3x^2 + x; -1 \leq x \leq 4$

   Extrema at:
   - $f(-1) = -4.7$
   - $f(4) = 0.8$ ABS MAX
   - $f(0.18) = 0.087$
   - $f(2.679) = -5.393$ ABS MIN

2. $f(x) = -21x^4 - 9x^3 + 50x^2 + 13x; -2 \leq x \leq 1.2$

   Extrema at:
   - $f(-2) = -90$ ABS MIN
   - $f(1.2) = 38.502$
   - $f(-1.204) = 28.408$
   - $f(0.127) = -0.832$
   - $f(1.01) = 33.01$ ABS MAX

3. A rectangle has its base on the $x$-axis and its two upper corners on the parabola $y = 12 - x^2$.

   a. Draw this scenario on the coordinate plane to the right, and draw a possible rectangle.

   b. Label the base and height of your rectangle in terms of $x$.

   c. Find the function $A(x)$ that represents the area of the rectangle.

   $A(x) = bh$
   $= 2x(12 - x^2)$
   $A(x) = 24x - 2x^3$

   d. What is the largest possible area of this rectangle?

   (Hint: Use a calculator to graph and find the maximum!)

   $A(x) = 32$ un^2

   e. At what $x$-value should the rectangle be drawn for the largest area?

   $x = 2$ units

4. Sketch (freehand) a graph of a function $g$ with domain all real numbers that satisfies all of the following conditions:

   ✓ a. There are no breaks in the graph (it is continuous).
   ✓ b. $g$ is decreasing on $(-\infty, -3)$ and on $(4, \infty)$
   ✓ c. $g$ is increasing on $(-1, 4)$
   ✓ d. $g(4) > g(-7)$
   ✓ e. $g(x) < 0$ on $(-4, 0)$
5. Mr. Sullivan has hired you to design a window in front of his house. His specifications are that it is to be a rectangular shape with a semi-circle on top (see figure) and the perimeter of the window is 288 inches. He wants you to create a window with the largest possible area that fits those specifications.

a. Using \( r \) as the radius of the semi-circle, label the top edge of the semi-circle in terms of \( r \). (Hint: What is the circumference of a circle?)

\[ C = 2\pi r, \text{ then } \frac{1}{2} C = \pi r \]

b. Label the bottom of the window in terms of \( r \).

c. Label the height of the rectangular portion as \( H \).

d. Find \( H \) in terms of \( r \).

\[
\text{Perimeter} = H + H + 2r + \pi r = 288 \\
2r + \pi r = 288 - 2r - \pi r \\
144 - \pi r = H
\]

e. Label the area of the semi-circle \( a_1(r) \). Find an equation for \( a_1(r) \).

\[
a_1(r) = \frac{1}{2} A_0 = \frac{1}{2} \pi r^2
\]

f. Label the area of the rectangle \( a_2(r) \). Find an equation for \( a_2(r) \).

\[
a_2(r) = b \cdot h = 2r \cdot H = 2r \left( 144 - r - \frac{\pi r}{2} \right)
\]

g. Find \( A(r) \), the total area of the window.

\[
A(r) = a_1(r) + a_2(r) = \left( \frac{1}{2} \pi r^2 \right) + \left( 288r - 2r^2 - \pi r^2 \right)
\]

\[
A(r) = -2r^2 + 288r - \frac{1}{2} \pi r^2
\]

h. What is the largest area of the window? (3 decimal places and use correct units)

\[
\text{\( \max A(r) \approx 5807.108 \text{ in}^2 \)}
\]

i. What is the width of the bottom of the window to create this large area? (3 decimal places and use correct units)

\[
r = 40.327138 \\
2r = 80.654 \text{ inches}
\]