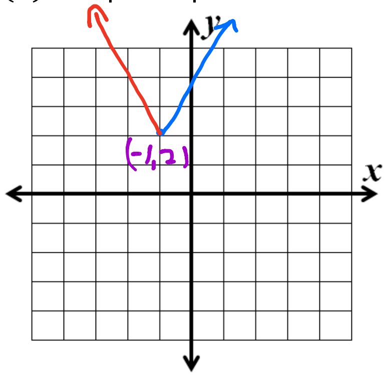
3.3 Application and Extension

1. Change the following absolute value function into a piecewise function by following the steps.

a) Graph $f(x) = 2|x+1| + 2$



b) Change the "absolute value" symbols to parentheses, and simplify the function

$f(x) = 2(x+1)+2$
$= 2x+2+2$
$f(x) = 2x+4$

This is the "positive slope" side of the function.

$m = 2$

c) Do the same thing you did in step *b* above, but change the slope to the opposite sign.

$f(x) = -2(x+1)+2$
$= -2x-2+2$
$f(x) = -2x$

$m = -2$

This is the "negative slope" side of the function.

d) Using steps *b-c* to help, write the function from step *a* as a piecewise function.

$$f(x) = \begin{cases} 2x+4 & \text{if } x \geq -1 \\ -2x & \text{if } x < -1 \end{cases}$$

2. Rewrite the function $f(x) = -\frac{5}{3}|x-3| + 4$ as a piecewise function and graph it.

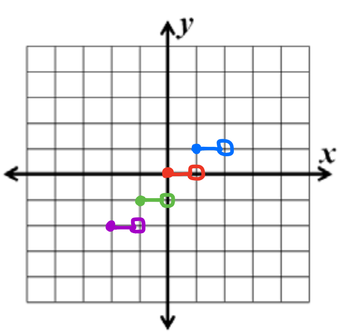
$f(x) = -\frac{5}{3}(x-3)+4$
$= -\frac{5}{3}x + 5 + 4$
$f(x) = -\frac{5}{3}x + 9$

$f(x) = \frac{5}{3}(x-3)+4$
$= \frac{5}{3}x - 5 + 4$
$f(x) = \frac{5}{3}x - 1$

$$f(x) = \begin{cases} -\frac{5}{3}x + 9 & \text{if } \text{if } x \geq 3 \\ \frac{5}{3}x - 1 & \text{if } x < 3 \end{cases}$$

3. The Greatest Integer Function (also called a "step" function) is modeled by the equation $f(x) = [\![x]\!]$. When you plug in a value for x, it returns the largest integer less than or equal to x. To the right is a table of a few input and output values. Finish the table and plot the points. Then graph and write a piecewise function for the domain $-2 \leq x < 2$.

x	*f(x)*
-2	-2
-1.78	-2
-1	-1
-0.5	-1
-0.01	-1
0.3	0
0.8	0
1	1
1.4	1
1.9999	1



$$f(x) = \begin{cases} -2 & \text{if } -2 \leq x < -1 \\ -1 & \text{if } -1 \leq x < 0 \\ 0 & \text{if } 0 \leq x < 1 \\ 1 & \text{if } 1 \leq x < 2 \end{cases}$$

What value(s) of k would make the following functions **continuous**.

4. $g(x) = \begin{cases} x+1, & x \le 2 \\ kx+6, & x > 2 \end{cases}$

$\boxed{@\ x=2}$

$x+1 = kx+6$

$(2)+1 = k(2)+6$

$3 = 2k+6$

$-3 = 2k$

$-\frac{3}{2} = k$

5. $h(x) = \begin{cases} 2x+3, & x < -1 \\ 7x-k, & x \ge -1 \end{cases}$

$\boxed{@\ x=-1}$

$2x+3 = 7x-k$

$2(-1)+3 = 7(-1)-k$

$-2+3 = -7-k$

$1 = -7-k$

$8 = -k$

$-8 = k$

6. $f(x) = \begin{cases} 3x^2-11x-4, & x \le 4 \\ kx^2-2x-1, & x > 4 \end{cases}$

$\boxed{@x=4}$

$3x^2-11x-4 = kx^2-2x-1$

$3(4)^2-11(4)-4 = k(4)^2-2(4)-1$

$= 16k-8-1$

$3(16)-44-4 = 16k-9$

$48-48 = 16k-9$

$0 = 16k-9$

$9 = 16k$

$\frac{9}{16} = k$

7. $w(x) = \begin{cases} -6x-12, & x < -3 \\ k^2-5k, & x = -3 \\ 6, & x > -3 \end{cases}$

$\boxed{@x=-3}$

$-6x-12 = k^2-5k = 6$

$-6(-3)-12 = k^2-5k = 6$

$18-12 = k^2-5k = 6$

$6 = k^2-5k = 6$

$k^2-5k = 6$

$k^2-5k-6 = 0$

$(k-6)(k+1) = 0$

$k-6=0 \quad k+1=0$

$k=6 \quad k=-1$

$k = -1, 6$

8. Kelly and Sullivan are planning their trip to the annual Star Trek convention. They need to rent a car to get there and find one car rental agency that charges $0.25 per mile if the total mileage does not exceed 100. If the total mileage exceeds 100, the agency charges $0.25 per mile for the first 100 miles and only $0.15 per mile for each mile over 100. If m represents the number of miles a rented vehicle is driven, express the mileage charge $C(m)$ as a function of m. Find $C(50)$ and $C(150)$, and graph C. (*This is not as easy as it first appears! The 2nd piece is challenging to figure out.)*

$C(m) = \begin{cases} 0.25m & \text{if } m \le 100 \\ 0.15(x-100)+25 & \text{if } m > 100 \end{cases}$

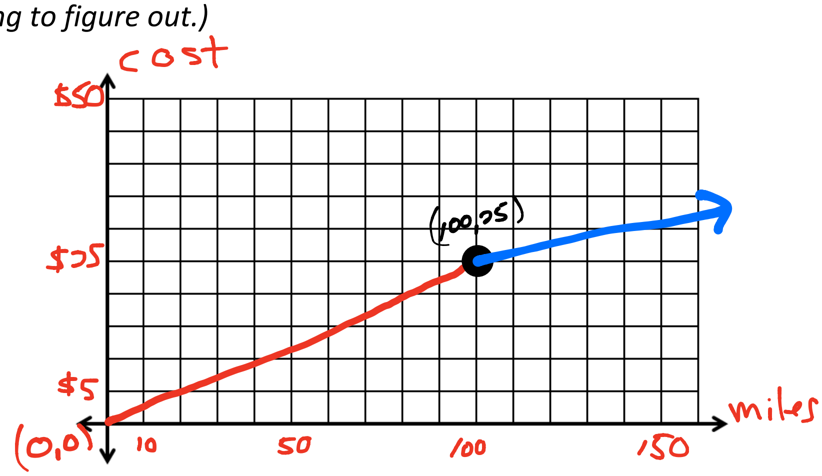
$C(50) = 0.25(50) = \$12.50$

$C(150) = 0.15[(150)-100]+25$

$= 0.15[50]+25$

$= 7.5+25$

$C(150) = \$32.50$



Explanation

If someone drives over 100 miles the first 100 miles will cost ($0.25)(100) = $25. Then, every mile over 100 is 15¢ per mile. So $0.15(\underbrace{100-x}_{\text{miles over }100})$ is the cost after you've paid $25 for the first 100 miles.

$\therefore 0.15(100-x)+25$ is cost over 100 miles.