Name: ______________________________ Date: ________________ Period: ________ **ID: 1**

Unit 3 REVIEW – Function Analysis

Pre-Calculus

Find the ***domain*** of the indicated function. Write your answers using inequality notation. ***Classify*** all discontinuities.

1. $f(x) = \frac{x}{x^2 - 9x} = \frac{\cancel{x}}{\cancel{x}(x-9)}$

$f(x) = \frac{1}{x-9}$

Denom ≠ 0

Cancel (Hole) $x \neq 0$

Stays (VA) $x - 9 \neq 0$, $x \neq 9$

DISC
Hole @ $x = 0$
VA @ $x = 9$

Domain: $\mathbb{R}$, $x \neq 0, 9$

2. $g(x) = \sqrt{16 - 4x}$

RADICAND ≥ 0

$16 - 4x \geq 0$

$-4x \geq -16$

$x \leq 4$

DOMAIN: $x \leq 4$

CONTINUOUS ON ITS DOMAIN

3. $h(t) = \frac{\sqrt{t+3}}{t-5}$

Denom ≠ 0

cancel (Hole)

Stays (VA)

$t - 5 \neq 0$

$t \neq 5$

RADICAND ≥ 0

$t + 3 \geq 0$

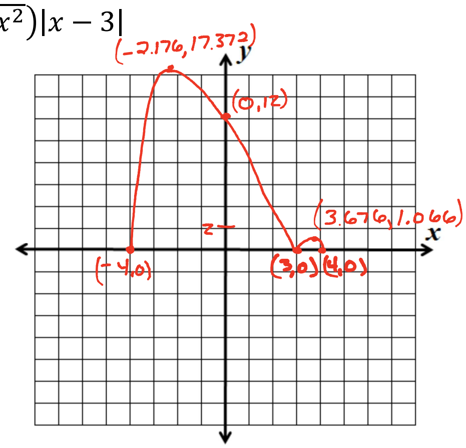
$t \geq -3$

DISC
VA @ $t = 5$

Domain: $t \geq -3$, $t \neq 5$

Domain:	Absolute max/min value(s):
$[-4, 4]$	ABS MAX: 17.372 ABS MIN: 0
Local extrema that are NOT absolute: LOCAL MAX: 1.066 LOCAL MIN: None	
Increasing: $(-4, -2.176) \cup (3, 3.676)$	Decreasing: $(-2.176, 3) \cup (3.676, 4)$
Left End-behavior: $\lim_{x \to -\infty} f(x) =$ DNE	Right End-behavior: $\lim_{x \to \infty} f(x) =$ DNE

4. $g(x) = \left(\sqrt{16 - x^2}\right)|x - 3|$



OPTIONAL WORK

RADICAND ≥ 0

$16 - x^2 \geq 0$

Boundary

$16 - x^2 = 0$

$16 = x^2$

$\pm 4 = x$

$16 - x^2 \geq 0$

$-$ | $+$ | $-$

-4 4

DOMAIN: $-4 \leq x \leq 4$

Find the value of the given function at the indicated domain value.

$$g(x) = \begin{cases} -2x^2 + 7x + 5, & x \leq 0 \\ 3 - x^3, & 2 < x < 8 \\ \sqrt{x + 17}, & x \geq 8 \end{cases} \qquad h(x) = \begin{cases} 5x^2 - 7x - 5, & x \leq -10 \\ x^3 - x, & -10 < x \leq 10 \\ 5x - |x - 25|, & x > 10 \end{cases}$$

5. $g(8) = \sqrt{(8) + 17}$

$= \sqrt{25}$

$g(8) = 5$

6. $h(-1) = (-1)^3 - (-1)$

$= -1 + 1$

$h(-1) = 0$

7. $h(10) = (10)^3 - (10)$

$= 1000 - 10$

$h(10) = 990$

8. $g(1) =$ DNE

Skillz Review: Solve or evaluate.

9. $\sqrt{-95} = \sqrt{-1 \cdot 5 \cdot 19}$

$= i\sqrt{95}$

10. $3x^2 = 24$

$x^2 = 8$

$x = \pm\sqrt{8}$

$x = \pm 2\sqrt{2}$

11. $-(x - 4)^2 - 5 = -54$

$-(x-4)^2 = -49$

$(x-4)^2 = 49$

$x - 4 = \pm 7$

$x = 4 \pm 7$

$x = -3, 11$

12. $3(x + 6)^2 + 20 = -28$

$3(x+6)^2 = -48$

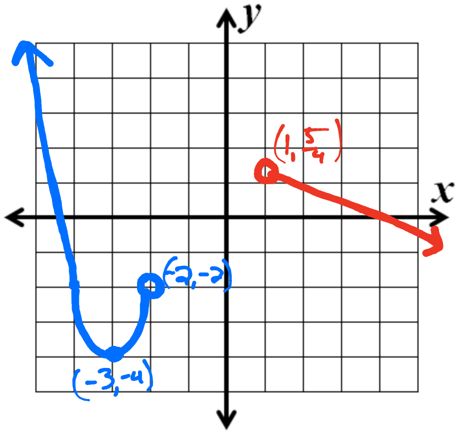
$(x+6)^2 = -16$

$x + 6 = \pm\sqrt{-16}$

$x = -6 \pm 4i$

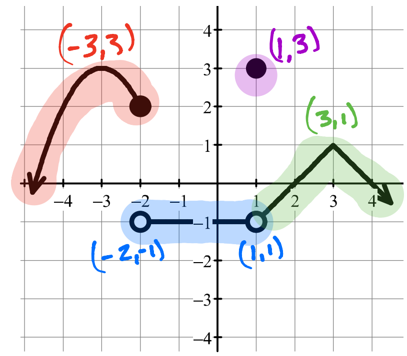
Graph the following piecewise functions.	**Given the graph of f, write out the function's equation. Use a linear expression ($mx + b$) for straight lines, absolute value if there is a "V" graph.**

13. $h(x) = \begin{cases} 2(x+3)^2 - 4, & x < -2 \\ -\frac{3}{4}x + 2, & x > 1 \end{cases}$



14.

$f(x) = \begin{cases} -(x+3)^2+3 & \text{if } x \le -2 \\ -1 & \text{if } -2 < x < 1 \\ 3 & \text{if } x = 1 \\ -|x-3|+1 & \text{if } x > 1 \end{cases}$



15. Is the following function continuous? (SHOW WORK!)

$f(x) = \begin{cases} -x^2 + 2x + 11, & x < -3 \\ 2x + 2, & x \ge -3 \end{cases}$

$-x^2+2x+11 = 2x+2$ $\boxed{@x=-3}$
$-(-3)^2+2(-3)+11 = 2(-3)+2$
$-(9)-6+11 = -6+2$
$-15+11 = -4$
$-4 = -4$

∴ CONTINUOUS

16. What value(s) of k would make the function continuous?

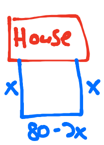
$g(x) = \begin{cases} -6x^2 + 18x, & x \le 1 \\ k^2 - k, & x > 1 \end{cases}$

$-6x^2+18x = k^2-k$ $\boxed{@x=1}$
$-6(1)^2+18(1) =$
$-6(1)+18 =$
$-6+18 =$
$12 = k^2-k$

$12 = k^2-k$
$0 = k^2-k-12$
$0 = (k-4)(k+3)$
$0 = k-4 \quad 0 = k+3$
$4 = k \quad -3 = k$

$k = -3, 4$

17. Mr. Kelly wants to create a rectangular feeding pen for his chickens, but only has 80 meters of fencing. He decides to use the side of his house as one side of the pen. $P = 80m$



a. If x represents the width of the pen, express its area A in terms of x. (The side of Kelly's house is the length.) $A(x) = (80-2x)x$

$A(x) = 80x - 2x^2$

b. What is the domain of the function A (determined by the physical restrictions)?

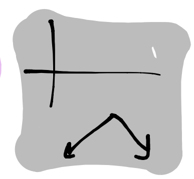
D: (0, 40)

18. Rewrite the function $f(x) = -\frac{3}{4}|x - 12| - 7$ as a piecewise function.

$f(x) = -\frac{3}{4}(x-12) - 7$
$= -\frac{3}{4}x + 9 - 7$
$f(x) = -\frac{3}{4}x + 2$

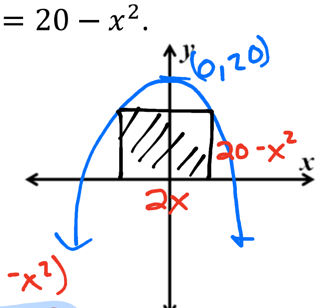
$f(x) = \frac{3}{4}(x-12) - 7$
$f(x) = \frac{3}{4}x - 9 - 7$
$f(x) = \frac{3}{4}x - 16$

$f(x) \begin{cases} -\frac{3}{4}x + 2 & \text{if } x > 12 \\ \frac{3}{4}x - 16 & \text{if } x \le 12 \end{cases}$



19. A rectangle has its base on the x-axis and its two upper corners on the parabola $y = 20 - x^2$.

a. Draw this scenario on the coordinate plane to the right, and draw one possible rectangle.
b. Label the base and height of your rectangle in terms of x.



c. Find the function $A(x)$ that represents the area of the rectangle.

$A(x) = b \cdot h$
$A(x) = 2x(20 - x^2)$
$A(x) = 40x - 2x^3$

d. What is the largest possible area of this rectangle?

68.853 un²

e. At what x-value should the rectangle be drawn for the largest area?

2.582 un