4.4 Inverse Functions

NOTES

Pre-Calculus

Write your questions here!

Inverse functions are reflections across line $y=x$ (y,x)

$f(x) = (x+2)^3 + 3$

$y = (x+2)^3 + 3$

$x = (y+2)^3 + 3$

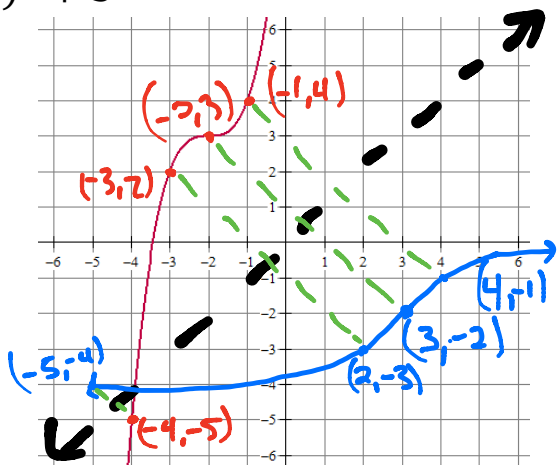
$x-3 = (y+2)^3$

$\sqrt[3]{x-3} = y+2$

$\sqrt[3]{x-3} - 2 = y$

x	$f(x)$
-2	3
-1	4
-3	2
-4	-5

x	$f^{-1}(x)$
3	-2
4	-1
2	-3
-5	-4



$f(x) = -2\sqrt{x-3} - 2$

$y = -2\sqrt{x-3} - 2$

$x = -2\sqrt{y-3} - 2$

$x+2 = -2\sqrt{y-3}$

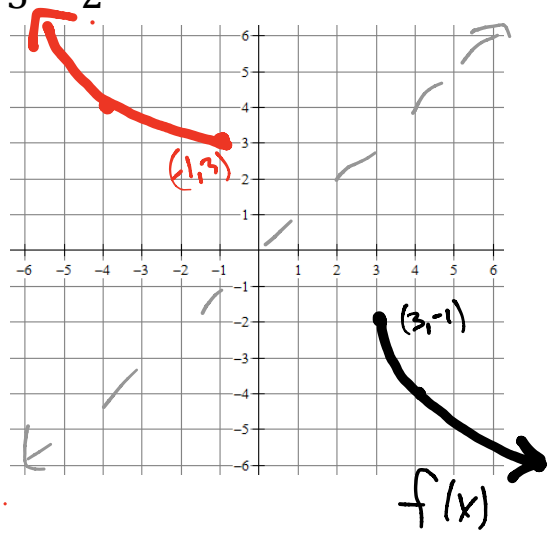
$\frac{x+2}{-2} = \sqrt{y-3}$

$\left(\frac{x+2}{-2}\right)^2 = y-3$

$\left(\frac{x+2}{-2}\right)^2 + 3 = y$

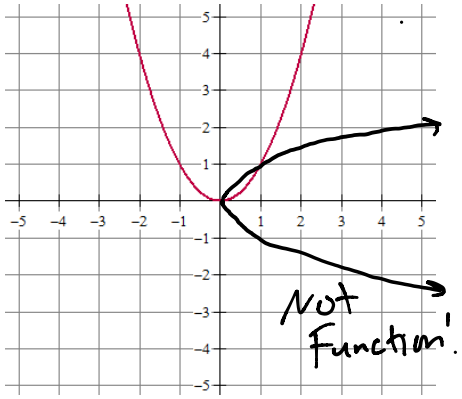
$f^{-1}(x) = \left(\frac{x+2}{-2}\right)^2 + 3$

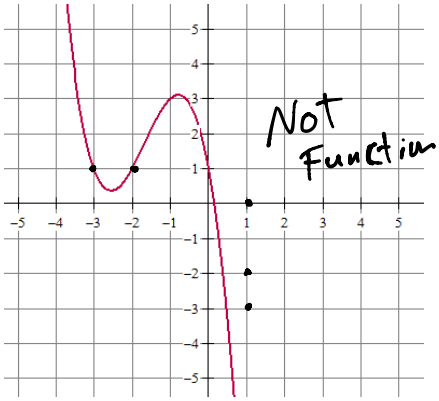
$f^{-1}(x) = \left[-\frac{1}{2}(x+2)\right]^2 + 3$



$f(x)$	$f(x)^{-1}$
DOMAIN: $[3,\infty)$	DOMAIN: $(-\infty,-2]$
RANGE: $(-\infty,-2]$	RANGE: $[3,\infty)$

Does the function have an inverse?

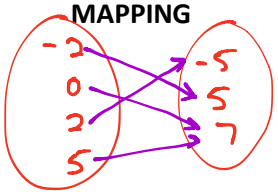




Horizontal line test

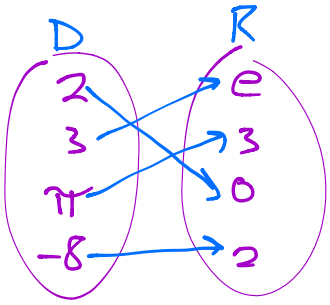
x	$f(x)$
-2	5
0	7
2	-5
5	7

MAPPING

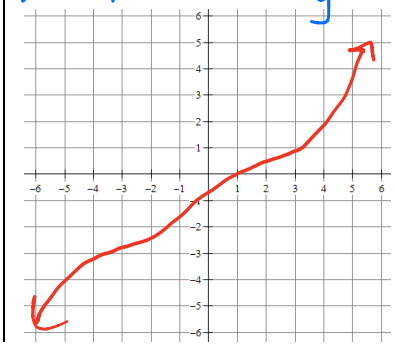


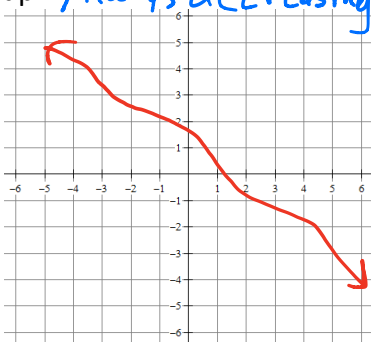
A **one-to-one function** passes the VLT and HLT.

1-1 MAPPING



Always increasing 1-1 Graph Always decreasing





PROVE 2 FUNCTIONS ARE INVERSES!

$f(g(x)) = f(g(x)) = x$

Are $f(x) = \frac{2}{x+3}$ and $g(x) = \frac{2}{x} - 3$ inverses?

$f(g(x)) = \frac{2}{g(x)+3}$

$= \frac{2}{\left(\frac{2}{x}-3\right)+3}$

$= \frac{2}{\frac{2}{x}}$

$f(g(x)) = x$

$g(f(x)) = \frac{2}{f(x)} - 3$

$= \frac{2}{\frac{2}{x+3}} - 3$

$= \frac{2(x+3)}{2} - 3$

$= x + 3 - 3$

$g(f(x)) = x$

$\therefore f$ and g are inverses

BRING THE PAIN

$y = \frac{2x-3}{x+4}$ FIND INVERSE

$x = \frac{2y-3}{y+4}$

$(y+4)x = 2y-3$

$xy + 4x = 2y - 3$

$4x + 3 = 2y - xy$

$4x + 3 = y(2-x)$

$\frac{4x+3}{2-x} = y$

$f^{-1}(x) = \frac{4x+3}{2-x}$

SUMMARY:

Now, summarize your notes here!