4.4 INVERSE FUNCTIONS APP

4.4 Inverse Functions

APPLICATION

1. Graph f and verify that f is one-to-one function. Find f^{-1} and add the graph of f^{-1} and the line $y = x$ to the graph f. State the domain and range of f and the domain and range of f^{-1}.

$$f(x) = (x+2)^3 - 1$$

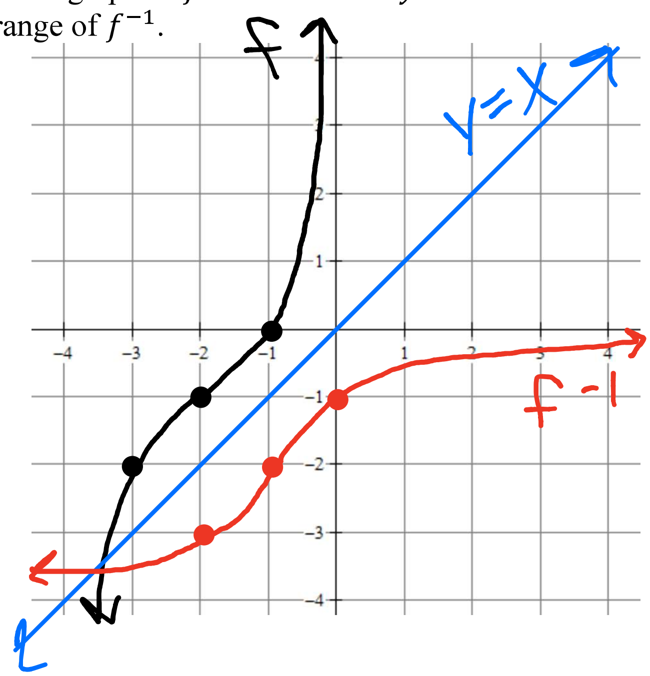
$x = (y+2)^3 - 1$

$x + 1 = (y+2)^3$

$\sqrt[3]{x+1} = y + 2$

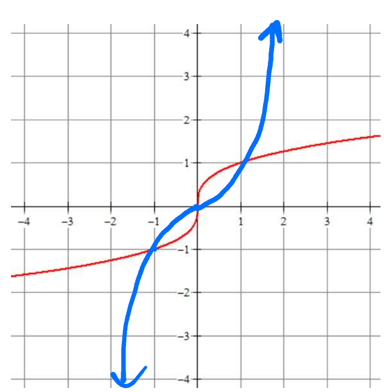
$\sqrt[3]{x+1} - 2 = y$

$f^{-1}(x) = \sqrt[3]{x+1} - 2$



	f	f^{-1}
D:	$(-\infty, \infty)$	$(-\infty, \infty)$
R:	$(-\infty, \infty)$	$(-\infty, \infty)$

2. The graph shows $f(x)$. On the same graph, sketch $f^{-1}(x)$.



3. Graph $f(x)=\dfrac{x-2}{x+2}$ in Y_1 of the graphing calculator with a "Standard Window". Find the $f^{-1}(x)$ and graph it in Y_2 of the graphing calculator. Graph $f(x) = x$ in Y_3 and us it to answer the following:

a. Explain why the functions are inverses according to your graph.

FIND f^{-1}

$$x=\frac{y-2}{y+2}$$
$$xy+2x=y-2$$
$$xy-y=-2x-2$$
$$y(x-1)=-2x-2$$
$$y=\frac{-2x-2}{x-1}$$

The two graphs are symmetric about the line $y=x$.

x	$f(x)$	$f^{-1}(x)$
5	.429	-3
-3	5	-1
-1	-3	0
0	-1	2
2	0	-6
-6	2	-1.429

b. Fill in the table and explain why they are inverses according to your table.

There are points whose inverses appear it the table. For example, $f(x)$ has $(-1,-3)$, $(0,-1)$, $(2,0)$ and $f^{-1}(x)$ has $(-3,-1)$, $(-1,0)$, $(0,2)$.

c. **BONUS CHALLENGE FOR THE ALGEBRA ROCK STARS ONLY!**
Algebraically prove $f(f^{-1}(x)) = x$

$$f(f^{-1}(x))=\frac{(f^{-1})-2}{(f^{-1})+2}=\frac{\left(\frac{-2x-2}{x-1}\right)-2}{\left(\frac{-2x-2}{x-1}\right)+2}=\frac{\left[\frac{(-2x-2)}{(x-1)}-\frac{2}{1}\right](x-1)}{\left[\frac{(-2x-2)}{(x-1)}+\frac{2}{1}\right](x-1)}=\frac{(-2x-2)-2(x-1)}{(-2x-2)+2(x-1)}=\frac{-2x-2-2x+2}{-2x-2+2x-2}$$
$$=\frac{-4x}{-4}$$
$$=x$$

Now repeat this for $f^{-1}(f(x))$:)

4. Complete the table of values given that f and g are inverse functions of each other.

x	$f(x)$	$g(x)$
0	5	2
1	3	4
2	0	5
3	9	1
4	1	7
5	2	0

5. What is unique about the inverse function of $f(x)=\dfrac{x}{x-1}$?

$$x=\frac{y}{y-1}$$
$$xy-x=y$$
$$xy-y=x$$
$$y(x-1)=x$$
$$y=\frac{x}{x-1}$$

$f(x)$ is its own inverse.

A function being its own inverse is like you being your own grandpa... just without the time travel ☮