

5.1 Practice – Operations on Polynomials

Name: _____

Pre-Calculus

Name the degree and leading coefficient.

1. $5a^2 - 2a^3 - 5a^{-2} - 5a^4$

Degree: 5
Leading Coefficient: -2

2. $8k^2 - 2 + k + 9k^3$

Degree: 3
Leading Coefficient: 9

3. $6x^4 + 5 - 9x^3 + 4$

Degree: 5
Leading Coefficient: 1

Simplify each expression.

4. $(7p^3 + 3p^2) - (p - 5p^3 - 2p^2)$
 $= 7p^3 + 3p^2 - p + 5p^3 + 2p^2$
 $= 12p^3 + 5p^2 - p$

5. $(6n^4 - 2n) + (3n^2 - 2n^4 - n)$
 $= 6n^4 - 2n + 3n^2 - 2n^4 - n$
 $= 4n^4 + 3n^2 - 3n$

6. $(2v^2 - 3v^4 + 4) + (1 - 2v^4 - 5v^2)$
 $= 2v^2 - 3v^4 + 4 + 1 - 2v^4 - 5v^2$
 $= -5v^4 - 3v^2 + 5$

7. $(3 + 4x^4) - (7x^4 + 3x^2 + 1) + (4x^4 + 3)$
 $= 3 + 4x^4 - 7x^4 - 3x^2 - 1 + 4x^4 + 3$
 $= x^4 - 3x^2 + 5$

8. $(8m + 6)(m^2 - 6m - 8)$
 $= 8m^3 - 48m^2 - 64m + 6m^2 - 36m - 48$
 $= 8m^3 - 42m^2 - 100m - 48$

9. $(5u^2 - 7v^2)(5u^2 + 7v^2)$
 $= 25u^4 - 49v^4$

mult $-210v^2$ | Add $11x$

10. $(8x - 5y^3)^2$
 $= 64x^2 - 80xy^3 + 25y^6$

mult $-90x^2$ | Add -9

11. $(6n^2 - 8n - 3)(8n - 6)$
 $= 48n^3 - 36n^2 - 64n^2 + 48n - 24n + 18$
 $= 48n^3 - 100n^2 + 24n + 18$

mult $-72p^2$ | Add $14p$

Factor each completely.

12. $7x^4 + 11x^3 - 30x^2$
 $= x^2(7x^2 + 11x - 30)$
 $= x^2[7x^2 + 21x + (-10x) - 30]$
 $= x^2[7x(x+3) - 10(x+3)]$
 $= x^2(x+3)(7x-10)$

13. $5x^2 - 9x - 18$
 $= 5x^2 - 15x + 6x - 18$
 $= 5x(x-3) + 6(x-3)$
 $= (x-3)(5x+6)$

14. $-9p^4 - 42p^3 + 72p^2$
 $= -3p^2[3p^2 + 14p - 24]$
 $= -3p^2[3p^2 + 18p - 4p - 24]$
 $= -3p^2[3p(p+6) - 4(p+6)]$
 $= -3p^2(p+6)(3p-4)$

15. $9x^2 - 16$
 $= (3x-4)(3x+4)$



16. $x^2 + 16$
 prime

17. $125w^2 - 80$
 $= 5(25w^2 - 16)$
 $= 5(5w-4)(5w+4)$



$$\begin{aligned}
 18. \quad & 5x^4 + 30x^2 - 200 \\
 & = 5 [x^4 + 6x^2 - 40] \\
 & = 5 [x^4 - 4x^2 + 10x^2 - 40] \\
 & = 5 [x^2(x^2 - 4) + 10(x^2 - 4)] \\
 & = 5 (x^2 - 4)(x^2 + 10) \\
 & = 5 (x - 2)(x + 2)(x^2 + 10)
 \end{aligned}$$

$$\begin{aligned}
 19. \quad & -x^4 + 3x^2 - 2 \\
 & = - (x^4 - 3x^2 + 2) \\
 & = - [x^4 - x^2 + (-2x^2) + 2] \\
 & = - [x^2(x^2 - 1) - 2(x^2 - 1)] \\
 & = - (x^2 - 1)(x^2 - 2) \\
 & = - (x - 1)(x + 1)(x^2 - 2)
 \end{aligned}$$

$$\begin{aligned}
 20. \quad & 30u^6 + 6u^3 - 108 \\
 & = 6(5u^6 + u^3 - 18) \\
 & = 6[5u^6 - 9u^3 + 10u^3 - 18] \\
 & = 6[u^3(5u^3 - 9) + 2(5u^3 - 9)] \\
 & = 6(5u^3 - 9)(u^3 + 2)
 \end{aligned}$$

mult $-90u^6$ | Add u^3

$$\begin{aligned}
 21. \quad & (42n^3 + 48n^2) + (49n - 56) \\
 & = 6n^2(7n + 8) - 7(7n + 8) \\
 & = (7n + 8)(6n^2 - 7)
 \end{aligned}$$

$$\begin{aligned}
 22. \quad & (12xy - 16x^2) + (21y - 28x) \\
 & = 4x(3y - 4x) + 7(3y - 4x) \\
 & = (3y - 4x)(4x + 7)
 \end{aligned}$$

$$\begin{aligned}
 23. \quad & (7k + 5k^3) + (-35k^2 - 49) \\
 & = k(7 + 5k^2) - 7(5k^2 + 7) \\
 & = (5k^2 + 7)(k - 7)
 \end{aligned}$$

Solve each equation using your new factoring abilities! Find ALL the solutions. Hint: don't forget to use the quadratic formula if a quadratic expression does not factor.

$$24. \quad x^3 + 2x^2 = -10x$$

$$x^3 + 2x^2 + 10x = 0$$

$$x(x^2 + 2x + 10) = 0$$

$x=0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(10)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{4 - 40}}{2}$$

$$x = \frac{-2 \pm \sqrt{-36}}{2}$$

$$x = \frac{-2 \pm 6i}{2}$$

$$x = -1 \pm 3i$$

$$x = 0, -1 \pm 3i$$

$$25. \quad x^3 = 6x^2 + x$$

$$x^3 - 6x^2 - x = 0$$

$$x(x^2 - 6x - 1) = 0$$

$x=0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{36 + 4}}{2}$$

$$x = \frac{6 \pm \sqrt{40}}{2}$$

$$x = \frac{6 \pm \sqrt{40}}{2}$$

$$x = 3 \pm \sqrt{10}$$

$$x = 0, 3 \pm \sqrt{10}$$

$$26. \quad x^6 - 5x^4 = x^2 - 5$$

$$x^6 - 5x^4 - x^2 + 5 = 0$$

$$x^4(x^2 - 5) - 1(x^2 - 5) = 0$$

$$(x^2 - 5)(x^4 - 1) = 0$$

$$(x^2 - 5)(x^2 - 1)(x^2 + 1) = 0$$

$$(x^2 - 5)(x - 1)(x + 1)(x^2 + 1) = 0$$

$$\begin{aligned}
 x^2 - 5 = 0 & \left\{ \begin{array}{l} x - 1 = 0 \\ x + 1 = 0 \end{array} \right. & \left\{ \begin{array}{l} x^2 + 1 = 0 \\ x^2 = -1 \end{array} \right. \\
 x^2 = 5 & \left\{ \begin{array}{l} x = 1 \\ x = -1 \end{array} \right. & \left\{ \begin{array}{l} x^2 = -1 \\ x = \pm i \end{array} \right. \\
 x = \pm \sqrt{5} & &
 \end{aligned}$$

$$x = \pm \sqrt{5}, \pm 1, \pm i$$

$$27. \quad x^4 + 6x^2 = 27$$

$$x^4 + 6x^2 - 27 = 0$$

$$(x^2 + 9)(x^2 - 3) = 0$$

$$x^2 + 9 = 0 \left\{ \begin{array}{l} x^2 - 3 = 0 \\ x^2 = -9 \\ x = \pm 3i \end{array} \right. \left\{ \begin{array}{l} x^2 = 3 \\ x = \pm \sqrt{3} \end{array} \right.$$

$$x = \pm 3i$$

$$x = \pm \sqrt{3}$$

$$x = \pm 3i, \pm \sqrt{3}$$

$$28. x^5 - x = 0$$

$$x(x^4 - 1) = 0$$

$$x(x^2 - 1)(x^2 + 1) = 0$$

$$x(x-1)(x+1)(x^2+1) = 0$$

$$x=0 \left\{ \begin{array}{l} x-1=0 \\ x=1 \end{array} \right\} \left\{ \begin{array}{l} x+1=0 \\ x=-1 \end{array} \right\} \left\{ \begin{array}{l} x^2+1=0 \\ x^2=-1 \\ x=\pm i \end{array} \right.$$

$$x = 0, \pm 1, \pm i$$

$$29. x^3 + 4x^2 + 3x = 0$$

$$x(x^2 + 4x + 3) = 0$$

$$x(x+3)(x+1) = 0$$

$$x=0 \left\{ \begin{array}{l} x+3=0 \\ x=-3 \end{array} \right\} \left\{ \begin{array}{l} x+1=0 \\ x=-1 \end{array} \right.$$

$$x = -3, -1, 0$$