Precalculus

Write your questions and thoughts here!

5.2 Poly Division and Factor Theorem

Name:_____

Long Division:

1. $(2x^4 + 3x - 1) \div (x^2 + 2x + 1)$

$2x^2 - 4x + 6 - \frac{5x+7}{x^2+2x+1}$

$x^2+2x+1 \overline{)\,2x^4+0x^3+0x^2+3x-1}$

$+(-2x^4-4x^3-2x^2)$

$-4x^3-2x^2+3x-1$

$-(+4x^3+8x^2+4x)$

$6x^2+7x-1$

$-(+6x^2+12x+6)$

$-5x-7$

2. $(4x^3 + 30x^2 + 74x + 64) \div (2x + 7)$

$2x^2+8x+9+\frac{1}{2x+7}$

$2x+7 \overline{)\,4x^3+30x^2+74x+64}$

$+(-4x^3-14x^2)$

$16x^2+74x+64$

$+(-16x^2-56x)$

$18x+64$

$+(-18x-63)$

1

Synthetic Substitution / Division:

3. $f(x) = 2x^3 - 5x^2 + 10$

$f(3) = 19$

3	2	-5	0	10
		6	3	9
	2	1	3	19

When can we use synthetic division?

when dividing by linear binomial

4. $f(x) = -5x^4 + x^3 - 3x$

$f(-2) = -82$

-2	-5	1	0	-3	0
		10	-22	44	-82
	-5	11	-22	41	-82

5. $(2n^4 - 10n^3 + 30n + 28) \div (n - 3)$

$= 2n^3 - 4n^2 - 12n - 6 + \frac{10}{n-3}$

$n-3=0$

$n=3$

3	2	-10	0	30	28
		6	-12	-36	-18
	2	-4	-12	-6	10 R

6. $(2x^2 - 5x - 12) \div (2x + 3)$

$= \frac{2x-8}{2} = x - 4$

$= 2(x+\frac{3}{2})$

$2x+3=0$

$2x=-3$

$x=-\frac{3}{2}$

$-\frac{3}{2}$	2	-5	-12
		-3	12
	2	-8	0

Remainder Theorem:

If a polynomial $f(x)$ is divided by $x - k$, then the remainder (r) is $r = f(k)$

Ex $f(x) \div (x+7)$ $\quad r = f(-7)$

Factor Theorem:

$(x - k)$ is a factor of the polynomial $f(x)$ if and only if $f(k) = 0$

Given one zero of the polynomial function, find the other zeros.

7. $f(x) = 2x^3 - 20x^2 + 66x - 72$;

3 is a zero

3	2	-20	66	-72
		6	-42	72
	2	-14	24	0

$f(x) = (x-3)(2x^2-14x+24)$

$0 = 2(x-3)(x^2-7x+12)$

$0 = 2(x-3)(x-3)(x-4)$

$0 \neq 2$

$0 = x-3$ $\quad x-3=0$ $\quad x-4=0$

$3 = x$ $\quad x=3$ $\quad x=4$

x-int = 3, 3, 4

8. $f(x) = 2x^3 - 17x^2 + 90x - 41$;

$\frac{1}{2}$ is a zero.

$\frac{1}{2}$	2	-17	90	-41
		1	-8	41
	2	-16	82	0

$f(x) = (x-\frac{1}{2})(x^2-8x+41)$

$0 = x - \frac{1}{2}$ $\quad 0 = x^2 - 8x + 41$

$\frac{1}{2} = x$ $\quad x = 4 \pm 5i$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-(-8) \pm \sqrt{(-8)^2-4(1)(41)}}{2(1)}$

$= \frac{8 \pm \sqrt{64-164}}{2}$

$= \frac{8 \pm \sqrt{-100}}{2}$

$= \frac{8 \pm 10i}{2}$

$= 4 \pm 5i$

Zeros = $\frac{1}{2}$, $4 \pm 5i$

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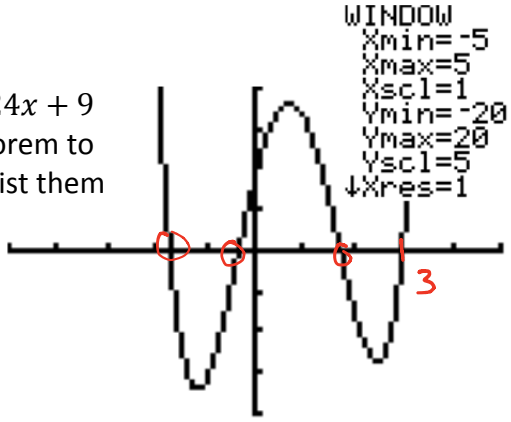
9. If $x = 3$ is a zero of $f(x)$, what else can we conclude?

x-3 is a factor of f(x)
x=3 is an x-intercept
x=3 is a solution to the equation f(x)=0

10. Given the graph of $f(x) = 3x^4 - 8x^3 - 12x^2 + 24x + 9$ to the right, use your knowledge of the factor theorem to find the exact values of all zeros of the function. List them from smallest to largest.



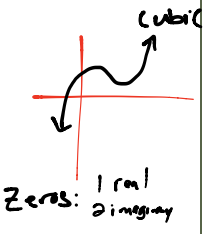
$$3 \mid 3 \quad -8 \quad -12 \quad 24 \quad 9$$
$$\quad\quad 9 \quad 3 \quad -27 \quad -9$$
$$\quad 3 \quad 1 \quad -9 \quad -3 \quad \boxed{0}$$

$(x-3)(3x^3+x^2-9x-3)=0$
$(x-3)[x^2(3x+1)-3(3x+1)]=0$
$(x-3)(3x+1)(x^2-3)=0$

$x-3=0 \Rightarrow x=3$
$3x+1=0 \Rightarrow 3x=-1 \Rightarrow x=-\frac{1}{3}$
$x^2-3=0 \Rightarrow x^2=3 \Rightarrow x=\pm\sqrt{3}$

Zeros: $-\sqrt{3}, -\frac{1}{3}, \sqrt{3}, 3$

The Fundamental Theorem of Algebra:



If $f(x)$ is a polynomial of degree n, then $f(x) = 0$ has exactly $\boldsymbol{n}$ solutions (both real and imaginary).

Imaginary zeros will come in _conjugate pairs_! In other words, each imaginary zero has a conjugate "buddy", so there won't ever be just one.

11. If $3 + 2i$ is a zero of a function, name one other zero.

$3-2i$

16 − i
16 + i

12. If $7i - \sqrt{5}$ is a zero of a function, name one other zero.

$-7i - \sqrt{5}$

Now summarize what you learned!

Skillz Review: Find the x- and y-intercepts for each function. SHOW ALL WORK!

1. $12x - 5y = 60$

x-int: y-int:

2. $f(x) = \frac{x^2+x-6}{4x-4}$

x-int: y-int: