

5.2 Practice – Poly Division and Factor Theorems

Name: _____

Pre-Calculus

For 1-3, use long division to divide the polynomials.

1. $(2x^2 - 7x + 4) \div (x - 2)$
 $= 2x - 3 + \frac{-2}{x-2}$

$$\begin{array}{r} 2x - 3 + \frac{-2}{x-2} \\ x-2 \overline{) 2x^2 - 7x + 4} \\ \underline{+ (-2x^2 + 4x)} \\ -3x + 4 \\ \underline{+ (+3x - 6)} \\ -2 \end{array}$$

2. $(y^2 - 9) \div (y + 1)$
 $= y - 1 + \frac{-8}{y+1}$

$$\begin{array}{r} y - 1 + \frac{-8}{y+1} \\ y+1 \overline{) y^2 + 0y - 9} \\ \underline{+ (-y^2 + 1y)} \\ -y - 9 \\ \underline{+ (+y + 1)} \\ -8 \end{array}$$

3. $(9x^5 - 3x^3 + 21x^2 - 2x + 4) \div (3x^2 + 1)$
 $= 3x^3 - 2x + 7 + \frac{-3}{3x^2+1}$

$$\begin{array}{r} 3x^3 - 2x + 7 + \frac{-3}{3x^2+1} \\ 3x^2+1 \overline{) 9x^5 + 0x^4 - 3x^3 + 21x^2 - 2x + 4} \\ \underline{+ (-9x^5 + 3x^3)} \\ -6x^3 + 21x^2 - 2x + 4 \\ \underline{+ (+6x^3 + 2x)} \\ 21x^2 - 4x + 4 \\ \underline{+ (-21x^2 + 7)} \\ -3 \end{array}$$

For 4-6, use synthetic division to divide the polynomials

4. $(x^2 + 3x - 3) \div (x - 3)$

$$= x + 6 + \frac{15}{x-3}$$

$$\begin{array}{r|rrr} 3 & 1 & 3 & -3 \\ & & 3 & 18 \\ \hline & 1 & 6 & 15 \end{array}$$

$(x-3)$

5. $(7x^6 + 50x^3 + 20x) \div (x + 2)$

$$= 7x^5 - 14x^4 + 28x^3 - 6x^2 + 12x - 4 + \frac{8}{x+2}$$

$$\begin{array}{r|rrrrrrr} -2 & 7 & 0 & 0 & 50 & 0 & 20 & 0 \\ & & -14 & 28 & -56 & 12 & -24 & 8 \\ \hline & 7 & -14 & 28 & -6 & 12 & -4 & 8 \end{array}$$

6. $(3x^4 - 4x^2 + 1) \div (x + 1)$

$$= 3x^3 - 3x^2 - x + 1$$

$$\begin{array}{r|rrrrr} -1 & 3 & 0 & -4 & 0 & 1 \\ & & -3 & 3 & 1 & -1 \\ \hline & 3 & -3 & -1 & 1 & 0 \end{array}$$

7. If $(x + 1)$ is a factor of $2x^5 + 2x^4 - 5x^3 - 5x^2 - 3x - 3$, what are all the factors of $f(x)$.

$$\begin{array}{r|rrrrrr} -1 & 2 & 2 & -5 & -5 & -3 & -3 \\ & & -2 & 0 & 5 & 0 & 3 \\ \hline & 2 & 0 & -5 & 0 & -3 & 0 \end{array}$$

$$2x^5 + 2x^4 - 5x^3 - 5x^2 - 3x - 3$$

$$= (x+1)(2x^4 - 5x^2 - 3)$$

$$= (x+1)[2x^4 - 6x^2 + x^2 - 3]$$

$$= (x+1)[2x^2(x^2-3) + 1(x^2-3)]$$

$$= (x+1)(x^2-3)(2x^2+1) \leftarrow \text{Answer}$$

8. Is $(x - 11)$ a factor of $(3x^4 - 33x^3 - 17x^2 + 187x - 11)$? I'm not quite sure. Lemme check.

$$\begin{array}{r|rrrrr} 11 & 3 & -33 & -17 & 187 & -11 \\ & & 33 & 0 & -187 & 0 \\ \hline & 3 & 0 & -17 & 0 & -11 \end{array}$$

NO, $(x-11)$ is not a factor.

$$\rightarrow 3(x + \frac{5}{3}) = 3(x - -\frac{5}{3})$$

9. If $(3x + 5)$ is a factor of $6x^3 + 31x^2 + 23x - 20$, what are all the factors of $f(x)$.

$$\begin{array}{r} \textcircled{1} \quad 3x+5 \overline{) 6x^3 + 31x^2 + 23x - 20} \\ \underline{-(6x^3 + 70x^2)} \\ 21x^2 + 23x - 20 \\ \underline{+(-21x^2 + 35x)} \\ -12x - 20 \\ \underline{+ (+12x + 20)} \\ 0 \end{array}$$

$$\begin{array}{l} \textcircled{2} \text{ FACTOR} \\ 2x^2 + 7x - 4 \\ = 2x^2 - x + 8x - 4 \\ = x(2x-1) + 4(2x-1) \\ = (2x-1)(x+4) \end{array}$$

$$\textcircled{3} \quad 6x^3 + 31x^2 + 23x - 20 = (3x+5)(2x-1)(x+4)$$

10. Is $(n + 1)$ a factor of $(6n^3 + 6n^2)$?

$$(n - -1)$$

$$\begin{aligned} f(n) &= 6n^3 + 6n^2 \\ f(-1) &= 6(-1)^3 + 6(-1)^2 \\ &= 6(-1) + 6(1) \\ &= -6 + 6 \\ f(-1) &= 0 \end{aligned}$$

\therefore Since $f(-1) = 0$, $(x+1)$ is a factor of $6n^3 + 6n^2$

For 11-14, a zero of the function is given. Find ALL the zeros of the function.

11. $f(x) = 4x^3 - 25x^2 - 154x + 40$; $f(10) = 0$

$$\textcircled{1} \quad 10 \left| \begin{array}{cccc} 4 & -25 & -154 & 40 \\ & 40 & 150 & -40 \\ \hline 4 & 15 & -4 & 0 \end{array} \right.$$

$\textcircled{2}$ FACTOR

$$4x^2 + 15x - 4 = 0$$

$$4x^2 - x + 16x - 4 = 0$$

$$x(4x-1) + 4(4x-1) = 0$$

$$(4x-1)(x+4) = 0$$

$$\left. \begin{array}{l} 4x-1=0 \\ 4x=1 \\ x=\frac{1}{4} \end{array} \right\} \begin{array}{l} x+4=0 \\ x=-4 \end{array}$$

$$\textcircled{3} \quad x = -4, \frac{1}{4}, 10$$

12. $f(x) = 3x^3 + 34x^2 + 72x - 64$; $f(-4) = 0$

$$\textcircled{1} \quad -4 \left| \begin{array}{cccc} 3 & 34 & 72 & -64 \\ & -12 & -88 & 64 \\ \hline 3 & 22 & -16 & 0 \end{array} \right.$$

$\textcircled{2}$ FACTOR

$$3x^2 + 22x - 16 = 0$$

$$3x^2 - 2x + 24x - 16 = 0$$

$$x(3x-2) + 8(3x-2) = 0$$

$$(3x-2)(x+8) = 0$$

$$\left. \begin{array}{l} 3x-2=0 \\ 3x=2 \\ x=\frac{2}{3} \end{array} \right\} \begin{array}{l} x+8=0 \\ x=-8 \end{array}$$

$$\textcircled{3} \quad x = -8, -4, \frac{2}{3}$$

13. $f(x) = x^5 + 2x^4 + 7x^3 + 14x^2 + 6x + 12$
 $f(-2) = 0$

$$\textcircled{1} \quad -2 \left| \begin{array}{cccccc} 1 & 2 & 7 & 14 & 6 & 12 \\ & -2 & 0 & -14 & 0 & -12 \\ \hline 1 & 0 & 7 & 0 & 6 & 0 \end{array} \right.$$

$\textcircled{2}$ FACTOR

$$x^4 + 7x^2 + 6 = 0$$

$$x^4 + 6x^2 + x^2 + 6 = 0$$

$$x^2(x^2+6) + 1(x^2+6) = 0$$

$$(x^2+6)(x^2+1) = 0$$

$$\left. \begin{array}{l} x^2+6=0 \\ x^2=-6 \\ x=\pm i\sqrt{6} \end{array} \right\} \begin{array}{l} x^2+1=0 \\ x^2=-1 \\ x=\pm i \end{array}$$

$$\textcircled{3} \quad x = \pm i\sqrt{6}, \pm i, -2$$

14. $f(x) = x^3 - 125$; $f(5) = 0$

$$\textcircled{1} \quad 5 \left| \begin{array}{cccc} 1 & 0 & 0 & -125 \\ & 5 & 25 & 125 \\ \hline 1 & 5 & 25 & 0 \end{array} \right.$$

$\textcircled{2}$ FACTOR DOESN'T

$$x^2 + 5x + 25 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(25)}}{2(1)}$$

$$= \frac{-5 \pm \sqrt{25 - 100}}{2}$$

$$= \frac{-5 \pm \sqrt{-75}}{2}$$

$$x = \frac{-5 \pm 5i\sqrt{3}}{2}$$

$$\textcircled{3} \quad x = 5, \frac{-5 \pm 5i\sqrt{3}}{2}$$

For 15-16, use the graph of the function to determine at least one zero, then find the exact values of all the zeros using the Factor Theorem.

15. $f(x) = 3x^4 + 16x^3 - 8x^2 - 112x - 91$

①
$$\begin{array}{r|rrrrr} -1 & 3 & 16 & -8 & -112 & -91 \\ & & -3 & -13 & 21 & 91 \\ \hline & 3 & 13 & -21 & -91 & \boxed{0} \end{array}$$

② **FACTOR**

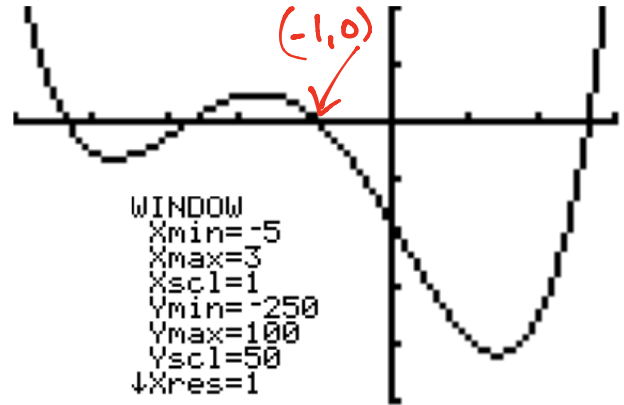
$$3x^3 + 13x^2 - 21x - 91 = 0$$

$$x^2(3x + 13) - 7(3x + 13) = 0$$

$$(x^2 - 7)(3x + 13) = 0$$

$$\left. \begin{array}{l} x^2 - 7 = 0 \\ x^2 = 7 \\ x = \pm\sqrt{7} \end{array} \right\} \begin{array}{l} 3x + 13 = 0 \\ 3x = -13 \\ x = -\frac{13}{3} \end{array}$$

③ $x = -\frac{13}{3}, \pm\sqrt{7}, -1$



16. $f(x) = 10x^3 - 31x^2 - 76x + 160$

①
$$\begin{array}{r|rrrr} 4 & 10 & -31 & -76 & 160 \\ & & 40 & 36 & -160 \\ \hline & 10 & 9 & -40 & \boxed{0} \end{array}$$

② **FACTOR**

$$10x^2 + 9x - 40 = 0$$

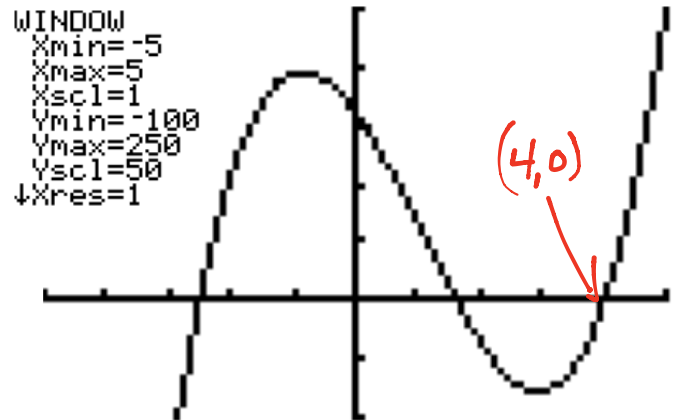
$$10x^2 - 16x + 25x - 40 = 0$$

$$2x(5x - 8) + 5(5x - 8) = 0$$

$$(5x - 8)(2x + 5) = 0$$

$$\left. \begin{array}{l} 5x - 8 = 0 \\ 5x = 8 \\ x = \frac{8}{5} \end{array} \right\} \begin{array}{l} 2x + 5 = 0 \\ 2x = -5 \\ x = -\frac{5}{2} \end{array}$$

③ $x = -\frac{5}{2}, \frac{8}{5}, 4$



For 17-20, one zero is given of $f(x)$. List one other zero.

17. $13 - 25i$

$13 + 25i$

18. $10i$

$-10i$

19. $3i + 1$

$-3i + 1$

20. $\sqrt{7} + 14i$

$\sqrt{7} - 14i$