Precalculus

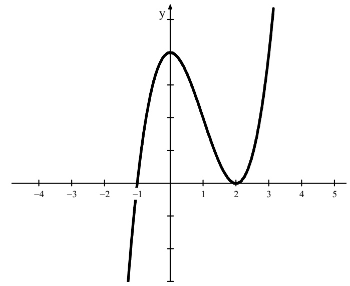
Write your questions and thoughts here!

NO CALC!

5.3 Polynomial Graphs

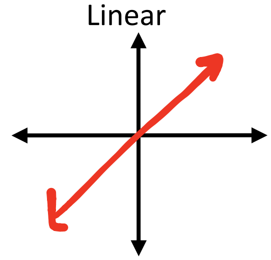
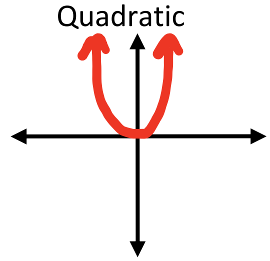
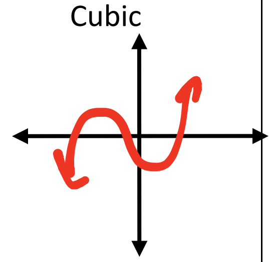
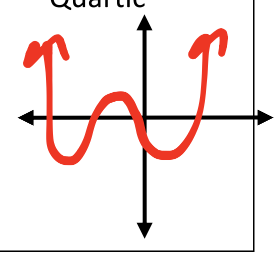
Name:________________________

Polynomial Graphs:

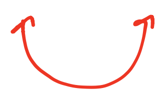
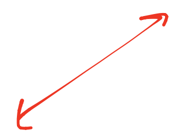
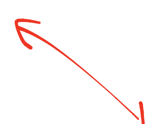


- There will be no discontinuities.
- There are no sharp corners.
- Domain: all real #.

Common Graphs:

Linear	Quadratic	Cubic	Quartic
			

End Behavior:

	Even Degree	ODD Degree
Positive Leading Coefficient	Ex $y=x^2$  $\lim_{x\to-\infty} f(x) = \infty$ $\lim_{x\to\infty} f(x) = \infty$	$y=x$  $\lim_{x\to-\infty} f(x) = -\infty$ $\lim_{x\to\infty} f(x) = \infty$
Negative Leading Coefficient	$y=-x^2$  $\lim_{x\to-\infty} f(x) = -\infty$ $\lim_{x\to\infty} f(x) = -\infty$	$y=-x$  $\lim_{x\to-\infty} f(x) = \infty$ $\lim_{x\to\infty} f(x) = -\infty$

Turning Points: (extrema)

A polynomial of degree n has **at most** $n-1$ turning points (extrema).

$y=x^2$ $y=x^3$

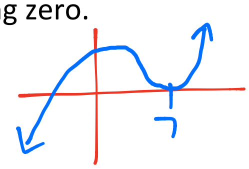
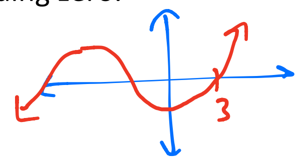
Zeros:

A polynomial of degree n has **at most** n real zeros (roots).

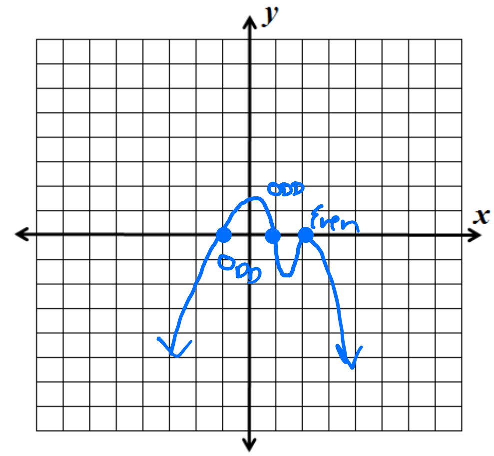
Multiplicity:

A factor's *multiplicity* is the number of times the factor occurs within the polynomial. For example, examine the function $f(x) = x^2(x-3)(x+1)^5(x-7)^8$. The factor $(x-3)$ has a multiplicity of 1, while the factor $(x-7)$ has a multiplicity of 8.

5.3 Polynomial Graphs

Even Multiplicity	______ Multiplicity
The graph is tangent to the x-axis at the corresponding zero.  $(x-7)^8$	The graph cross the x-axis at the corresponding zero.  $(x-3)$

1. $f(x) = -2(x+1)(x-2)^2(x-1)^3$



a. How does the graph behave with relation to the x-axis at $x = 2$? Tangent

b. What are the real zeros of the function?
$-1, 1, 2$

c. What is the degree of the function?
6

d. Describe the end behavior using limit notation.
$\lim_{x\to-\infty} f(x) = -\infty$ $\lim_{x\to-\infty} f(x) = -\infty$

e. Sketch a possible graph.

2. Factor the function $f(x) = x^5 + 6x^4 - 8x^3 - 96x^2 - 128x$ and sketch the graph if $f(-2) = 0$.

$$\begin{array}{r|rrrrrr} -2 & 1 & 6 & -8 & -96 & -128 & 0 \\ & & -2 & -8 & 32 & 128 & 0 \\ \hline & 1 & 4 & -16 & -64 & 0 & \boxed{0} \end{array}$$

$f(x) = (x+2)(x^4+4x^3-16x^2-64x)$

$0 = (x+2)\,x\,[x^3+4x^2-16x-64]$

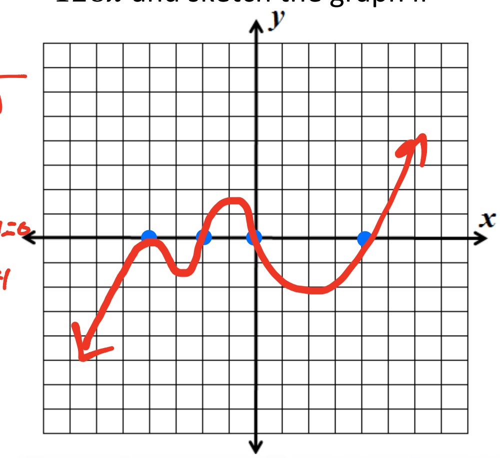
$= x(x+2)[x^2(x+4)-16(x+4)]$

$= x(x+2)(x+4)(x^2-16)$

$= x(x+2)(x+4)(x-4)(x+4)$

$0 = x(x+2)(x+4)^2(x-4)$

$x=0$ $x+2=0$, $x=-2$ $x+4=0$, $x=-4$ $x-4=0$, $x=4$



3. Given the graph of $g(x)$ on the right, identify the following:

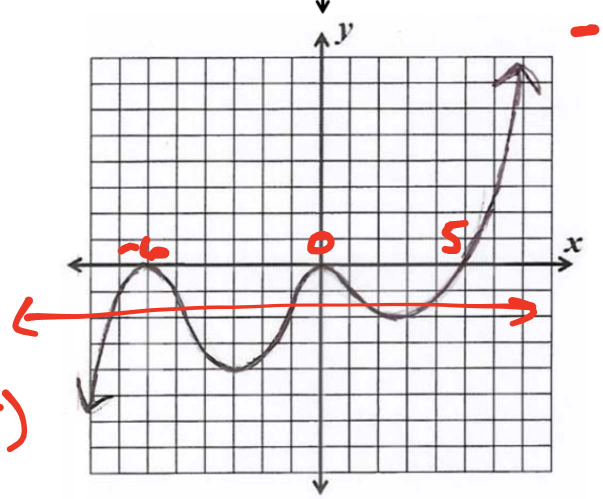
a. Local minimum value(s) $=-4, -2$

b. Local maximum value(s) $= 0$

c. Minimum Degree $= 5$

d. Write out a possible function. Leave it in factored form.

$f(x) = x^2(x+6)^2(x-5)$



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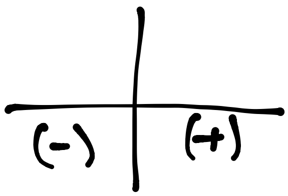

Descartes' Rule of Signs:

The # of positive real zeros	=	The # of changes in sign of the coefficients of f(x).	(*or less than this by an even number*)
The # of negative real zeros	=	The # of changes in sign of the coefficients of f(-x).	(*or less than this by an even number*)

4. List the possible numbers of zeros for f.

$f(x) = 2x^6 - 3x^5 + 7x^4 - x^2 - 2x - 11$

1 1 1 3 sign changes

Possible number of ***positive*** zeros: 3, 1

$f(-x) = 2x^6 + 3x^5 + 7x^4 - x^2 + 2x - 11$

3 sign changes

Possible number of ***negative*** zeros: 3, 1

Now summarize what you learned!