Name: ________________________ Date:________________ Period: ________ **ID: 1**

Unit 5 REVIEW – Polynomials

Pre-Calculus

1. $h(t) = -5t + 7 - 8t^3 + 9t^2$

Degree: 3

Leading Coefficient: -8

2. Simplify $(4 - 8x)(4x^2 + x - 7) - (x - 9x^3)$

$= 16x^2 + 4x - 28 - 32x^3 - 8x^2 + 56x - x + 9x^3$

$= -23x^3 + 8x^2 + 59x - 28$

Factor each completely:

3. $75a^2 - 108$

$= 3(25a^2 - 36)$

$= 3(5a-6)(5a+6)$

4. $6x^2 - 37xy + 45y^2$

$= 6x^2 - 10xy + {}^-27xy + 45y^2$

$= 2x(3x-5y) - 9y(3x-5y)$

$= (3x-5y)(2x-9y)$

5. $-x^4 - 12x^2 - 32$

$= -(x^4 + 12x^2 + 32)$

$= -(x^2+4)(x^2+8)$

Solve each of the equations. Give exact answers and find ALL solutions (real and imaginary).

6. $x^3 + 4x^2 = 38x$

$x^3 + 4x^2 - 38x = 0$

$x(x^2 + 4x - 38) = 0$

$x = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(-38)}}{2(1)}$

$x = \frac{-4 \pm \sqrt{16 + 152}}{2}$

$x = \frac{-4 \pm \sqrt{168}}{2}$

$x = \frac{-4 \pm 2\sqrt{42}}{2}$

$x = \frac{2(-2 \pm \sqrt{42})}{2}$

$x = -2 \pm \sqrt{42}$

$x = 0, -2+\sqrt{42}, -2-\sqrt{42}$

7. $x^3 + 9x^2 = 4x^2 - x - 5$

$x^3 + 5x^2 + x + 5 = 0$

$x^2(x+5) + 1(x+5) = 0$

$(x+5)(x^2+1) = 0$

$x+5=0$, $x=-5$

$x^2+1=0$, $x^2=-1$, $x = \pm i$

$x = -5, i, -i$

8. $x^5 = 5x - 4x^3$

$x^5 + 4x^3 - 5x = 0$

$x(x^4 + 4x^2 - 5) = 0$

$x(x^2+5)(x^2-1) = 0$

$x(x^2+5)(x-1)(x+1) = 0$

$x = 0$

$x^2+5=0$, $x^2=-5$, $x = \pm i\sqrt{5}$

$x-1=0$, $x=1$

$x+1=0$, $x=-1$

$x = 0, \pm i\sqrt{5}, \pm 1$

9. Use long division to divide $(1 + 4x^3 - 8x^2)$ by $(2x + 2)$.

$2x^2 - 6x + 6 + \frac{-11}{2x+2}$

$2x+2 \overline{)\, 4x^3 - 8x^2 + 0x + 1}$

$+(-4x^3 + {}^-4x^2)$

$-12x^2 + 0x + 1$

$+(+12x^2 + 12x)$

$12x + 1$

$+(-12x + {}^-12)$

-11

10. Is $(x + 7)$ a factor of $(x^5 + 4x^4 - 21x^3 + 2x + 14)$? Show any work that leads you to your conclusion.

$(x-(-7))$

$f(-7) = (-7)^5 + 4(-7)^4 - 21(-7)^3 + 2(-7) + 14$

$= -16807 + 4(2401) - 21(-343) - 14 + 14$

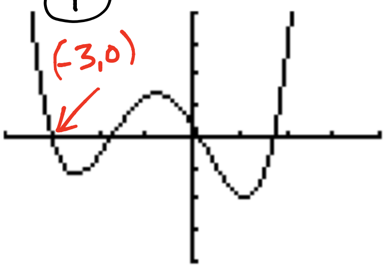
$= -16807 + 9604 + 7203 + 0$

$f(-7) = 0$

Yes, since $f(-7) = 0$, $(x+7)$ is a factor of $(x^5 + 4x^4 - 21x^3 + 2x + 14)$

11. Use the graph of the function to determine at least one zero, then find the exact values of all the zeros using the Factor Theorem. $f(x) = 7x^4 + 20x^3 - 24x^2 - 60x + 9$

① $(-3, 0)$



WINDOW
Xmin=-4
Xmax=4
Xscl=1
Ymin=-100
Ymax=100
Yscl=25
↓Xres=1

②
$$\begin{array}{r|rrrrr} -3 & 7 & 20 & -24 & -60 & 9 \\ & & -21 & 3 & 63 & -9 \\ \hline & 7 & -1 & -21 & 3 & \boxed{R0} \end{array}$$

③
$7x^3 - x^2 - 21x + 3 = 0$

$x^2(7x-1) - 3(7x-1) = 0$

$(7x-1)(x^2-3) = 0$

$7x - 1 = 0$ | $x^2 - 3 = 0$

$7x = 1$ | $x^2 = 3$

$x = 1/7$ | $x = \pm\sqrt{3}$

④ $x = -3, \pm\sqrt{3}, 1/7$

12. List ALL the zeros of $f(x) = x^3 - 1$ given that $f(1) = 0$.

①
$$\begin{array}{r|rrrr} 1 & 1 & 0 & 0 & -1 \\ & & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & \boxed{R0} \end{array}$$

② $x^2 + x + 1 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-(1) \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$

$x = \dfrac{-1 \pm \sqrt{1-4}}{2}$

$x = \dfrac{-1 \pm \sqrt{-3}}{2}$

$x = \dfrac{-1 \pm i\sqrt{3}}{2}$

③ $x = 1, \dfrac{-1 \pm i\sqrt{3}}{2}$

13. If $7i - 6$ is a zero of $f(x)$, list one other zero.

$-7i - 6$

14. If $\sqrt{17} - 4i$ is a zero of $f(x)$, list one other zero.

$\sqrt{17} + 4i$

15. Factor the function $f(x) = -x^4 - 4x^3 + 16x^2 + 64x$ and sketch the graph. (zeros and end behavior are vital)

$f(x) = -x[x^3 + 4x^2 - 16x - 64]$

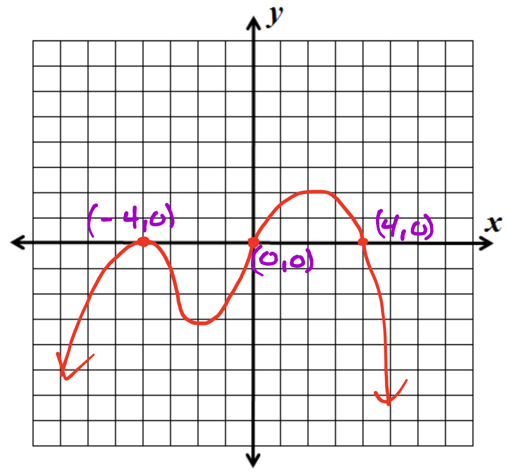
$f(x) = -x[x^2(x+4) - 16(x+4)]$

$f(x) = -x(x+4)(x^2-16)$

$f(x) = -x(x+4)(x+4)(x-4)$

$f(x) = -x(x+4)^2(x-4)$

End



For 16-17, list the possible numbers of positive real zeros and negative real zeros.

16. $f(x) = 3x^6 + 7x^3 - 2x - 6$

Possible # of positive = 1

Possible # of negative = 3 or 1

$f(-x) = 3(-x)^6 + 3(-x)^3 - 2(-x) - 6$

$= 3x^6 + 3(-x^3) + 2x - 6$

$= 3x^6 - 3x^3 + 2x - 6$

17. $h(x) = 5x^6 + 2x^3 - 36x - 9$

Possible # of positive = 1

Possible # of negative = 3 or 1

$h(-x) = 5(-x)^6 + 2(-x)^3 - 36(-x) - 9$

$= 5x^6 + 2(-x^3) + 36x - 9$

$= 5x^6 - 2x^3 + 36x - 9$

18. Given the graph of $g(x)$, identify the following:

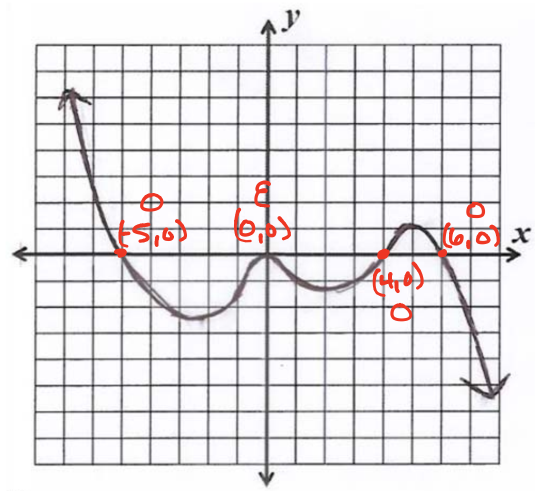
a. Local minimum value(s) $-2.5, -1.2$

b. Local maximum value(s) $0, 1$

c. Minimum Degree 5

d. Sign of leading coefficient. negative

e. Write out a possible function for the graph. Leave it in factored form. $y=-x^2(x+5)(x-4)(x-6)$



19. Scientists and fishermen often estimate the weight of a fish from its length. The data in the table give the average weight of North American sturgeon for certain lengths. Because weight is associated with volume, which involves three dimensions, we might expect that weight would be associated with the cube of the length.

a) Find a cubic model from the data.

$y = .005x^3 - .117x^2 + 1.425x - 5.004$

b) Use the model to estimate the weight of a sturgeon of length 56 inches.

632.031 oz

c) Compare the weight of a sturgeon of length 44 inches as given by Table 1 with the weight given by the model. Table = 282 oz MODEL = 279.42 oz

North American Sturgeon

Length (in.)	Weight (oz.)
18	13
22	26
26	46
30	75
34	115
38	166
44	282
52	492
60	796

Source: www.thefishernet.com

20. Sketch (freehand) a graph of a function f that satisfies all of the following conditions:

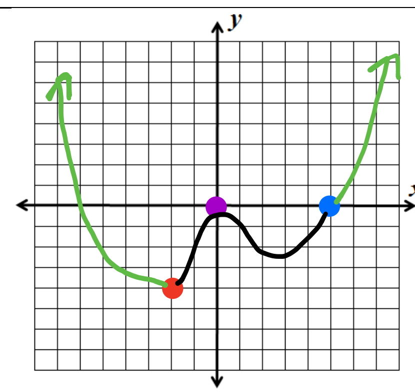
a. $f(-2) = -4$

b. $(x-5)$ is a factor of $f(x)$ and has a multiplicity of 7.

c. The leading coefficient is positive

d. x^2 is a factor of $f(x)$.

e. $f(x)$ is even.



21. A rectangular container measuring 1 foot by 2 feet by 4 feet is covered with a layer of lead shielding of uniform thickness (see the figure).

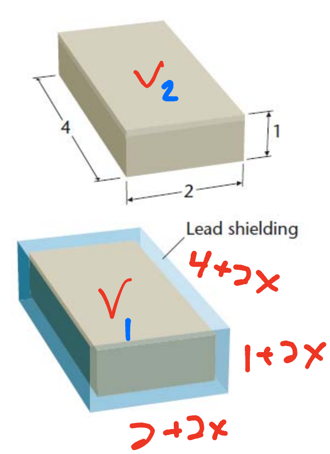
a) Find the volume of lead shielding V as a function of the thickness x (in feet) of the shielding.

$V_s = V_1 - V_2$

$V_s = A_{box_1} h_1 - A_{box_2} h_2$

$V_s = (2+2x)(1+2x)(4+2x) - 4 \cdot 2 \cdot 1$

$V_s = (2+2x)(1+2x)(4+2x) - 8$



b) Find the volume of the lead shielding if the thickness of the shielding is 0.05 feet.

$V(.05) = (2+2(.05))(1+2(.05))(4+2(.05)) - 8$

$= (2+.1)(1+.1)(4+.1) - 8$

$= (2.1)1.1(4.1) - 8$

$= 9.471 - 8$

$V(.05) = 1.471 \text{ ft}^3$