6.1 Solving Rational Equations

PRACTICE

Directions: Solve each equation. Check for extraneous solutions.

1) $\frac{2a-12}{a+5} = -2$

$(a+5)$ $(a+5)$

$2a-12 = -2a-10$

$4a-12 = -10$

$4a = 2$

$a = \frac{2}{4}$

$a = 1/2$

Denom $\neq 0$

$a+5 \neq 0$

$a \neq -5$

2) $\frac{1}{x+4} = \frac{1}{x^2+4x} + \frac{5x-6}{x^2+4x}$

$x(x+4)$ $x(x+4)$ $x(x+4)$

$x = 1 + 5x - 6$

$x = -5 + 5x$

$-4x = -5$

$x = 5/4$

Denom $\neq 0$

$x(x+4) \neq 0$

$x \neq 0$ $\quad x+4 \neq 0$

$x \neq -4$

3) $\frac{1}{h+3} + \frac{2}{n} = \frac{3}{h+3}$

$(h+3)h$ $(h+3)h$ $(h+3)h$

$n + 2(h+3) = 3h$

$h + 2h + 6 = 3h$

$3h + 6 = 3h$

$6 \neq 0$

NO Solution

4) $\frac{2-n-3n^2}{3n-2} = n+2$

$\frac{-(3n^2+n-2)}{3n-2} = n+2$

$\frac{-[3n^2+3n+-2n-2]}{3n-2} = n+2$

$\frac{-[3n(n+1)-2(n+1)]}{3n-2} = n+2$

$\frac{-(n+1)(3n-2)}{3n-2} = n+2$

$-n-1 = n+2$

$-1 = 2n+2$

$-3 = 2n$

$-3/2 = n$

Denom $\neq 0$

$3n-2 \neq 0$

$3n \neq 2$

$n \neq 2/3$

5) $\frac{1}{c^2-c-12} - \frac{1}{c+3} = \frac{6}{c^2-c-12}$

$(c-4)(c+3)$ $(c-4)(c+3)$ $(c-4)(c+3)$

$1 - (c-4) = 6$

$1 - c + 4 = 6$

$-c + 5 = 6$

$-c = 1$

$c = -1$

Denom $\neq 0$

$(c-4)(c+3) \neq 0$

$c-4 \neq 0$ $\quad c+3 \neq 0$

$c \neq 5$ $\quad c \neq -4$

6) $\frac{6x}{x+4} + 4 = \frac{2x+2}{x-1}$

$(x+4)(x-1)$ $(x+4)(x-1)$ $(x+4)(x-1)$

$6x(x-1) + 4(x+4)(x-1) = (2x+2)(x+4)$

$6x^2 - 6x + 4(x^2+3x-4) = 2x^2+10x+8$

$6x^2 - 6x + 4x^2 + 12x - 16 = 2x^2 + 10x + 8$

$10x^2 + 6x - 16 = 2x^2 + 10x + 8$

$8x^2 - 4x - 24 = 0$

$4[2x^2 - x - 6] = 0$

$4[(2x^2 - 4x) + (3x - 6)] = 0$

$4[2x(x-2) + 3(x-2)] = 0$

$4(x-2)(2x+3) = 0$

$4 \neq 0$ $\quad x-2 = 0$ $\quad 2x+3 = 0$

$x = 2$ $\quad 2x = -3$

$x = -3/2$

$x = -3/2, 2$

Denom $\neq 0$

$x+4 \neq 0$ $\quad x-1 \neq 0$

$x \neq -4$ $\quad x \neq 1$

7) $\frac{5x}{x-2} = 7 + \frac{10}{x-2}$

$5x = 7(x-2) + 10$

$5x = 7x - 14 + 10$

$5x = 7x - 4$

$-2x = -4$

$x = 2$

Denom $\neq 0$

$x - 2 \neq 0$

$x \neq 2$

NO SOLUTION

8) $\frac{2}{3x} + \frac{2}{3} = \frac{8}{x+6}$

$2(x+6) + 2x(x+6) = 8 \cdot 3 \cdot x$

$2x + 12 + 2x^2 + 12x = 24x$

$2x^2 + 14x + 12 = 24x$

$2x^2 - 10x + 12 = 0$

$2(x^2 - 5x + 6) = 0$

$2(x-3)(x-2) = 0$

$2 \neq 0 \quad x-3=0 \quad x-2=0$

$x = 3 \quad x = 2$

Denom $\neq 0$

$3x \neq 0 \quad x+6 \neq 0$

$x \neq 0 \quad x \neq -6$

$x = 2, 3$

9) $\frac{1}{a+1} + \frac{1}{a-1} = \frac{2}{a^2-1}$

$(a-1) + (a+1) = 2$

$2a = 2$

$a = 1$

Denom $\neq 0$

$a+1 \neq 0 \quad a-1 \neq 0$

$a \neq -1 \quad a \neq 1$

NO solution

10) $\frac{1}{2} - \frac{x-3}{2} = \frac{x-4}{2x-10}$

$(x-5) - (x-5)(x-3) = x-4$

$(x-5) - (x^2 - 8x + 15) = x-4$

$x - 5 - x^2 + 8x - 15 = x - 4$

$-x^2 + 9x - 20 = x - 4$

$-x^2 + 8x - 16 = 0$

$-(x^2 - 8x + 16) = 0$

$-(x-4)^2 = 0$

$(x-4)^2 = 0$

$x - 4 = 0$

$x = 4$

Denom $\neq 0$

$2 \neq 0 \quad x-5 \neq 0$

$x \neq 5$

11) $\frac{1}{a^2} + \frac{5a^2+14a-24}{a^2} = \frac{a-5}{a}$

$1 + (5a^2 + 14a - 24) = a(a-5)$

$5a^2 + 14a - 23 = a^2 - 5a$

$4a^2 + 19a - 23 = 0$

$(4a^2 - 4a) + (23a - 23) = 0$

$4a(a-1) + 23(a-1) = 0$

$(a-1)(4a+23) = 0$

$a - 1 = 0 \quad 4a + 23 = 0$

$a = 1 \quad 4a = -23$

$a = -23/4$

$a = -23/4, 1$

Denom $\neq 0$

$a^2 \neq 0$

$a \neq 0$

12) $\frac{x+2}{x^2-3x-4} - \frac{1}{x^2-3x-4} = \frac{x-6}{x-4}$

$\frac{(x+2)}{(x-4)(x+1)} - \frac{(1)}{(x-4)(x+1)} = \frac{(x-6)(x+1)}{x-4}$

$(x+2) - (1) = (x-6)(x+1)$

$x + 1 = x^2 - 5x - 6$

$0 = x^2 - 6x - 7$

$0 = (x-7)(x+1)$

$0 = x - 7 \quad 0 = x + 1$

$7 = x \quad -1 = x$

Denom $\neq 0$

$x - 4 \neq 0 \quad x + 1 \neq 0$

$x \neq 4 \quad x \neq -1$

$x = 7$