Evaluate each limit using direct substitution. You might have to factor the function to evaluate the limit.

#1) $\lim\limits_{x\to 2}(3x^2+8x-13) = 3(2)^2+8(2)-13$

$= 3(4)+16-13$

$= 12+3$

$\lim\limits_{x\to 2} f(x) = 15$

#2) $\lim\limits_{x\to 3}\frac{2x+6}{x} = \frac{2(3)+6}{3}$

$= \frac{6+6}{3}$

$= \frac{12}{3}$

$\lim\limits_{x\to 3} f(x) = 4$

Evaluate each limit. You might have to factor the function to evaluate the limit.

#1) $\lim\limits_{x\to -4}\frac{x^2+6x+8}{x+4} = \lim\limits_{x\to -4}\frac{(x+2)\cancel{(x+4)}}{\cancel{x+4}}$

$= \lim\limits_{x\to -4}(x+2)$

$= (-4)+2$

$\lim\limits_{x\to -4} f(x) = -2$

#2) $\lim\limits_{x\to 0}\frac{2x^3+5x^2+4x}{x} = \lim\limits_{x\to 0}\frac{\cancel{x}(2x^2+5x+4)}{\cancel{x}}$

$= \lim\limits_{x\to 0}(2x^2+5x+4)$

$= 2(0)^2+5(0)+4$

$\lim\limits_{x\to 0} f(x) = 4$

Find the excluded value of the function. Then evaluate the limit of the function at the excluded value. You might have to factor the function to evaluate the limit or you may have to do something much more sinister.

#1) $f(x)=\frac{\sqrt{x}-10}{x-100}$

Denom $\neq 0$

$\boxed{\begin{aligned} x-100 &\neq 0 \\ x &\neq 100 \end{aligned}}$

$\lim\limits_{x\to 100}\frac{\sqrt{x}-10}{x-100}\cdot\frac{\sqrt{x}+10}{\sqrt{x}+10} = \lim\limits_{x\to 100}\frac{\cancel{x-100}}{\cancel{(x-100)}(\sqrt{x}+10)}$

$= \lim\limits_{x\to 100}\frac{1}{\sqrt{x}+10}$

$= \frac{1}{\sqrt{100}+10}$

$= \frac{1}{10+10}$

$\lim\limits_{x\to 100} f(x) = \frac{1}{20}$

#2) $f(x)=\frac{\sqrt{x}-1}{x-1}$

Denom $\neq 0$

$\boxed{\begin{aligned} x-1 &\neq 0 \\ x &\neq 1 \end{aligned}}$

$\lim\limits_{x\to 1}\frac{\sqrt{x}-1}{x-1}\cdot\frac{\sqrt{x}+1}{\sqrt{x}+1} = \lim\limits_{x\to 1}\frac{\cancel{x-1}}{\cancel{(x-1)}(\sqrt{x}+1)}$

$= \lim\limits_{x\to 1}\frac{1}{\sqrt{x}+1}$

$= \frac{1}{\sqrt{1}+1}$

$= \frac{1}{1+1}$

$\lim\limits_{x\to 1} f(x) = \frac{1}{2}$

The derivative is defined as $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$. Find the derivative of each function below.

#1) $f(x) = x^2 - 3x - 7$

$$f'(x) = \lim_{h\to 0} \frac{[(x+h)^2 - 3(x+h) - 7] - [x^2 - 3x - 7]}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2} + 2hx + h^2 - \cancel{3x} - 3h - \cancel{7} - \cancel{x^2} + \cancel{3x} + \cancel{7}}{h}$$

$$= \lim_{h\to 0} \frac{2hx + h^2 - 3h}{h}$$

$$= \lim_{h\to 0} \frac{h(2x + h - 3)}{h}$$

$$= \lim_{h\to 0} (2x + h - 3)$$

$$= 2x + (0) - 3$$

$$f'(x) = 2x - 3$$

#2) $f(x) = 3x^2 - 6x - 10$

$$f'(x) = \lim_{h\to 0} \frac{[3(x+h)^2 - 6(x+h) - 10] - [3x^2 - 6x - 10]}{h}$$

$$= \lim_{h\to 0} \frac{3(x^2 + 2hx + h^2) - 6x - 6h - 10 - 3x^2 + 6x + 10}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{3x^2} + 6hx + 3h^2 - \cancel{6x} - 6h - \cancel{10} - \cancel{3x^2} + \cancel{6x} + \cancel{10}}{h}$$

$$= \lim_{h\to 0} \frac{6hx + 3h^2 - 6h}{h}$$

$$= \lim_{h\to 0} \frac{h(6x + 3h - 6)}{h}$$

$$= \lim_{h\to 0} (6x + 3h - 6)$$

$$= 6x + 3(0) - 6$$

$$f'(x) = 6x - 6$$

#3) $f(x) = \frac{x}{x+1}$

$$f'(x) = \lim_{h\to 0} \frac{\left[\frac{x+h}{(x+h)+1}\right] - \left[\frac{x}{x+1}\right]}{h} \quad (x+1)(x+h+1)$$

$$= \lim_{h\to 0} \frac{(x+h)(x+1) - x(x+h+1)}{h(x+1)(x+h+1)}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2} + \cancel{hx} + \cancel{x} + h - \cancel{x^2} - \cancel{hx} - \cancel{x}}{h(x+1)(x+h+1)}$$

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(x+1)(x+h+1)}$$

$$= \lim_{h\to 0} \frac{1}{(x+1)(x+h+1)}$$

$$= \frac{1}{(x+1)(x+(0)+1)}$$

$$= \frac{1}{(x+1)(x+1)}$$

$$f'(x) = \frac{1}{(x+1)^2}$$