

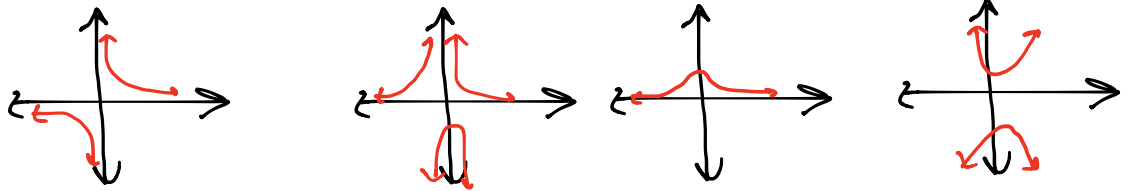
6.3 Graphing Rational Functions

Write your questions here!

Rational Equations: *Function with polynomial divided by polynomial.*

$$f(x) = \frac{p(x)}{q(x)}$$

Sketch some of them!



Find the Domain

Removable (Holes)

Cancelled factors

Vertical Asymptote(s)

uncanceled denom factor

$y = \frac{x-4}{2x^2-8x}$ $= \frac{x-4}{2x(x-4)}$ $= \frac{1}{2x}$ <p><i>Removable</i> $x-4=0$ $x=4$</p> <p><i>VA</i> $2x=0$ $x=0$</p> <p>$\therefore D = \mathbb{R}, x \neq 0, 4$</p>	$y = \frac{6x^2+7x-3}{x^2+4}$ $= \frac{(2x+3)(3x-1)}{x^2+4}$ <p><i>VA</i> $x^2+4=0$ $x^2=-4$ $x=\pm 2i$</p> <p>$\therefore D = \mathbb{R}$</p>	$y = \frac{x^2+5}{x+1}$ <p><i>VA</i> $x+1=0$ $x=-1$</p> <p>$\therefore D = \mathbb{R}, x \neq -1$</p>
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Find the x- and y-intercept(s)

Y-intercept (x = 0)

X-intercept (y = 0)

Simplify first

$y = \frac{x-4}{2x^2-8x} = \frac{x-4}{2x(x-4)}$ $y = \frac{1}{2x}$ <p><i>X-int = none</i> $0 \neq \frac{1}{2x}$ $0 \neq 1$</p> <p><i>Y-int = none</i> $y = \frac{1}{2(0)}$ $y = \text{undef}$</p> <p><i>NO intercepts</i></p>	$y = \frac{6x^2+7x-3}{x^2+4} = \frac{(2x+3)(3x-1)}{x^2+4}$ <p><i>X-int = $-\frac{3}{2}, \frac{1}{3}$</i> $0 = (2x+3)(3x-1)$ $0 = 2x+3 \quad 3x-1=0$ $-3=2x \quad 3x=1$ $-\frac{3}{2}=x \quad x=\frac{1}{3}$</p> <p><i>Y-int = $-\frac{3}{4}$</i> $y = \frac{6(0)^2+7(0)-3}{(0)^2+4}$ $y = \frac{-3}{4}$</p>	$y = \frac{x^2+5}{x+1}$ <p><i>X-int = none</i> $0 = x^2+5$ $-5 = x^2$ $i\sqrt{5} = x$</p> <p><i>Y-int = 5</i> $y = \frac{(0)^2+5}{(0)+1}$ $y = 5$</p>
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Horizontal Asymptotes

Degree in the numerator is bigger: *NO HA*

Degree in the denominator is bigger: *HA @ y=0*

Degrees are the same: *HA @ y = $\frac{a}{b}$ (a and b are lead coefficients)*

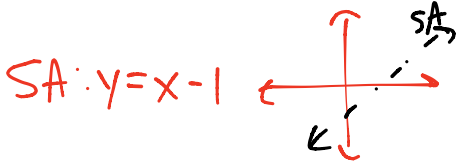
$y = \frac{x-4}{2x^2-8x}$ <p><i>HA: y=0</i></p>	$y = \frac{6x^2+7x-3}{x^2+4}$ <p><i>y = $\frac{6}{1}$</i> <i>HA: y=6</i></p>	$y = \frac{x^2+5}{x+1}$ <p><i>NO HA.</i></p>
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Slant (Oblique) Asymptotes

SA exists if degree of numerator is one more than degree of denominator
 Use long division to find it. (Discard remainder)

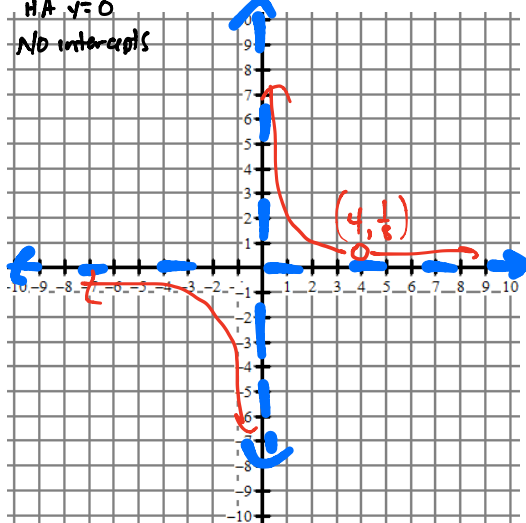
$$y = \frac{x^2 + 5}{x + 1}$$

$$\begin{array}{r} x-1 + \frac{6}{x+1} \\ x+1 \overline{) x^2 + 0x + 5} \\ \underline{-(x+1)} \\ -x+5 \\ \underline{+(x+1)} \\ 6 \end{array}$$



Hole $x=4$
 VA $x=0$
 HA $y=0$
 No intercepts

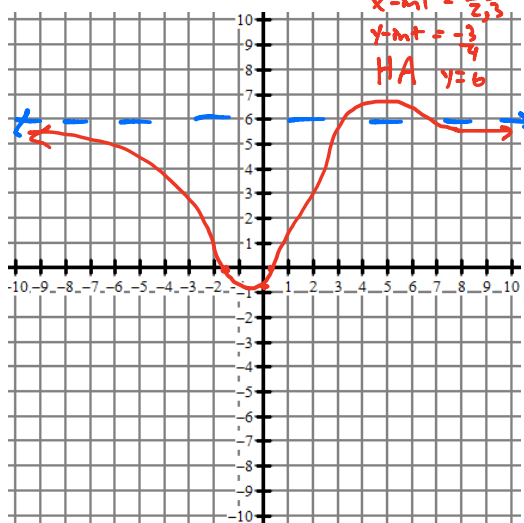
$$y = \frac{x-4}{2x^2-8x} = \frac{1}{2x} \cdot \text{hole}(4, \frac{1}{8})$$



Put it all together

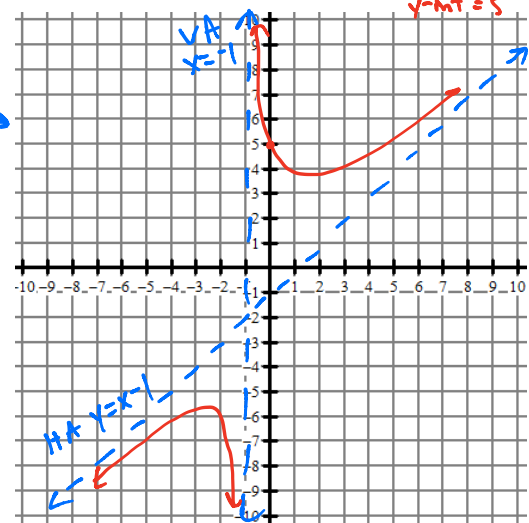
$$y = \frac{6x^2+7x-3}{x^2+4}$$

D: \mathbb{R}
 $x\text{-int} = -\frac{3}{2}, \frac{1}{3}$
 $y\text{-int} = -\frac{3}{4}$
 HA $y=6$



$$y = \frac{x^2+5}{x+1}$$

VA: $x=-1$
 SA: $y=x-1$
 No x-int
 $y\text{-int} = 5$



USE CALC

$$y = \frac{2x^3 - x^2 - x}{x^2 + 2x + 1} = \frac{x(2x^2 - x - 1)}{(x+1)^2} = \frac{x(x-1)(2x+1)}{(x+1)^2}$$

Hole/Vertical Asymptotes:

No Holes
 VA: $x=-1$
 $(x+1)^2 = 0$
 $x+1=0$
 $x=-1$
 D: $\mathbb{R}, x \neq -1$

Y-int: 0

$$y = \frac{2(0)^3 - (0)^2 - (0)}{(0)^2 + 2(0) + 1} = \frac{0}{1} = 0$$

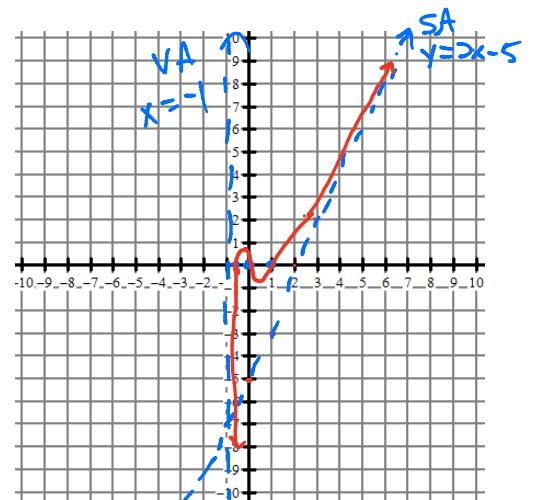
X-int: $-\frac{1}{2}, 0, 1$

$$0 = x \left\{ \begin{array}{l} 0 = 2x+1 \\ -1 = 2x \end{array} \right. \left\{ \begin{array}{l} 0 = x-1 \\ 1 = x \end{array} \right.$$

Horizontal/Slant Asymptote:

SA: $y=2x-5$

$$\begin{array}{r} 2x-5, 27x+5 \\ x^2+2x+1 \overline{) 2x^3 - x^2 - x + 0} \\ \underline{-(2x^3 + 4x^2 + 2x)} \\ -5x^2 - 3x \\ \underline{+(5x^2 + 10x + 5)} \\ 7x + 5 \end{array}$$



You try!

$$y = \frac{x^2 - 4x - 12}{x^2 + 5x + 6} = \frac{(x-6)(x+2)}{(x+2)(x+3)} = \frac{x-6}{x+3}, \text{ hole } (-2, -8)$$

Hole/Vertical Asymptotes:

Hole $(-2, -8)$

$$\begin{array}{l} x+2=0 \\ x=-2 \end{array} \quad \begin{array}{l} y = \frac{(-2)-6}{(-2)+3} \\ y = -\frac{8}{1} \\ y = -8 \end{array}$$

VA: $x = -3$

$$\begin{array}{l} x+3=0 \\ x=-3 \end{array}$$

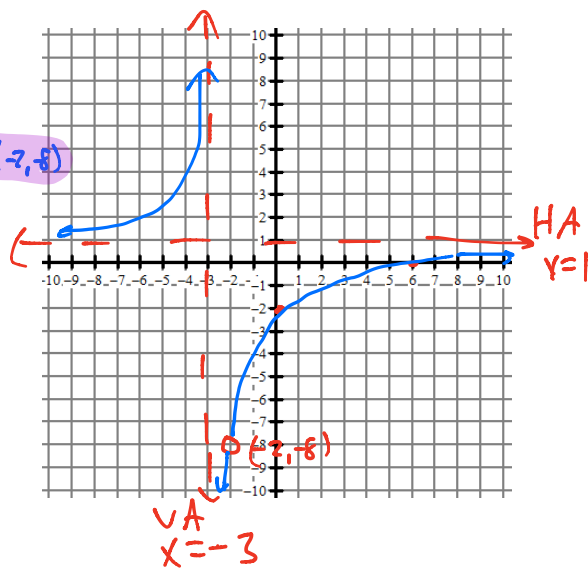
Y-Int: -2

$$y = \frac{(0)-6}{(0)+3} \\ y = -2$$

X-int: 6

$$\begin{array}{l} 0 = x-6 \\ 6 = x \end{array}$$

Horizontal/Slant Asymptote: HA $y = \frac{1}{1}$
HA $y = 1$



SUMMARY:

Now, summarize your notes here!

REVIEW SKILLZ

Directions: Simplify. Use only positive exponents.

1) $5x^{-4}(6x^{-3})$

2) $\frac{15y^7}{20y^{14}}$

3) $(4h^{-5})^3$

4) $\left(\frac{3n^{13} \cdot 4m^{-8}n^{-5}}{(m^3n^{-3})^2}\right)^{-3}$