

6.3 Graphing Rational Functions

APPLICATION

1) $y = \frac{4x+5}{x+1}$

Hole/Vertical Asymptotes:

no Hole VA @ $x = -1$

$$\begin{aligned} x+1 &= 0 \\ x &= -1 \end{aligned}$$

Y-Int: 5

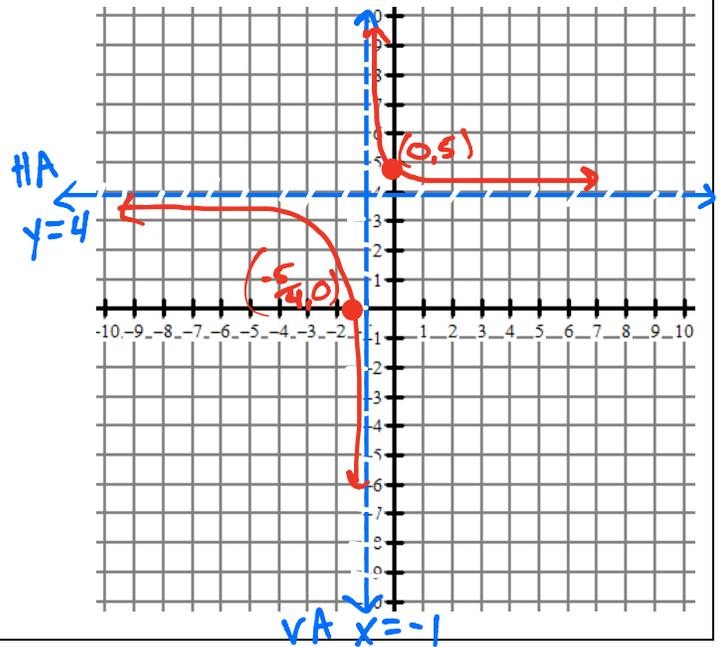
X-int: $-5/4$

$$y = \frac{4(0)+5}{(0)+1} = \frac{5}{1} = 5$$

$$\begin{aligned} 0 &= 4x+5 \\ -5 &= 4x \\ -5/4 &= x \end{aligned}$$

Horizontal/Slant Asymptote:

HA @ $y = 4$



2) Consider the rational function: $y = \frac{x^4 - 4x^2 + 1}{x^2 - 1} = \frac{x^4 - 4x^2 + 1}{(x-1)(x+1)}$

Find any holes or vertical asymptotes:

no Holes

VA @ $x = -1, 1$

$$\begin{aligned} x-1 &= 0 & x+1 &= 0 \\ x &= 1 & x &= -1 \end{aligned}$$

What is the y-intercept?

y-int = 1

$$\begin{aligned} y &= \frac{(0)^4 - 4(0)^2 + 1}{(0)^2 - 1} \\ y &= \frac{1}{-1} \\ y &= -1 \end{aligned}$$

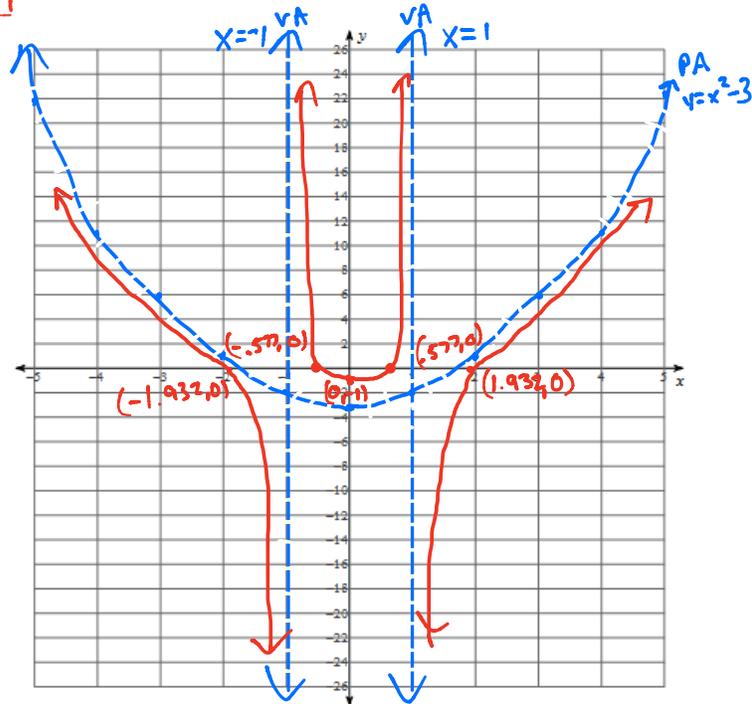
What are the x-intercepts?

$$0 = x^4 - 4x^2 + 1$$

Doesn't factor :(

Use CALC to find zeros

$$x \approx -0.577, 0.577, -1.932, 1.932$$



Since the degree of the numerator is greater than the degree of the denominator there are no horizontal asymptotes. Perform polynomial long division to find the asymptote. (Remember to eliminate the remainder)

$$\begin{array}{r} x^2 - 3 \\ x^2 - 1 \overline{) x^4 + 0x^3 - 4x^2 + 0x + 1} \\ \underline{+ (-x^4 + x^2)} \\ -3x^2 + 1 \\ \underline{+ (+3x^2 - 3)} \\ \text{Remainder} \end{array}$$

What is the equation of the asymptote? Is it linear? What shape would you describe it as? Draw this on the graph.

$y = x^2 - 3$ It is a parabolic asymptote.

Finish sketching the graph. Use the window that shows the domain of $[-5, 5]$ and range of $[-25, 25]$