6.4 Variation and Modeling

As the number of rooms **increases**, the total cost of the job *increases*

As the number of rooms **doubles**, the total cost of the job *doubles*

As the number of rooms is **halved**, the total cost of the job *is halved*

Number of Rooms	Cost of the Job
1	$20
2	$40
3	$60
4	$80
5	$100

DIRECT VARIATION

y is directly proportional to x means $y = kx$

k is called "constant of proportionality"

Ex 1: Y varies directly with x. If y= -4 when x = 2, find y, when x= -6.

$$y = kx$$
$$-4 = k(2)$$
$$-2 = k$$

$$y = kx$$
$$y = (-2)(-6)$$
$$y = 12$$

Ex 2: The force, F, exerted by a spring is directly proportional to the distance, d, that it is stretched. Find the constant of proportionality and the equation of variation if F = 12 pounds when d = 1/3 foot.

$$F = kd$$
$$12 = k\left(\tfrac{1}{3}\right)$$
$$36 = k$$

$$F = 36d$$

As the speed **increases**, the time taken *decreases*

As the speed **doubles**, the time taken is *halved*

As the speed is **halved**, the time taken is *doubled*

Speed, x (km/h)	Time taken, y (in hours)
20	6
30	4
40	3
60	2
120	1

INVERSE VARIATION

y is inversely proportional to x means $y = \dfrac{k}{x}$

k is called "constant of proportionality"

Ex 3: Y varies inversely with x. If y= 40 when x= 16, find x when y= -5

$$y = \frac{k}{x}$$
$$40 = \frac{k}{16}$$
$$640 = k$$

$$-5 = \frac{k}{x}$$
$$-5 = \frac{640}{x}$$
$$-5x = 640$$
$$x = -128$$

Ex 4: The note played by each pipe in a pipe organ is determined by the frequency of vibration of the air in the in the pipe. The fundamental frequency, F, of vibration of air in an organ pipe is inversely proportional to the length, L, of the pipe. Find the fundamental frequency of a 1.6 foot pipe if the fundamental frequency of an 8-foot pipe is 64 vibrations per second.

$$F = \frac{K}{L}$$

$$64 = \frac{k}{8}$$

$$512 = K$$

$$F = \frac{K}{L}$$

$$F = \frac{512}{1.6}$$

$$F = 320 \text{ vibrations per second}$$

JOINT VARIATION

w is jointly proportional to x and y means $w = Kxy$

K is called "constant of proportionality"

Ex 5: The volume, V, of a right circular cone is jointly proportional to the square of its radius, r, and its height, h. Find the equation of variation if a cone of height 8 inches and radius 3 inches has a volume of 24π cubic inches.

$$V = Kr^2 h$$

$$24\pi = K(3^2)(8)$$

$$24\pi = 72K$$

$$\frac{\pi}{3} = K$$

$$V = \frac{1}{3}\pi r^2 h$$

Ex 6: The frequency, f, of a vibrating guitar string is directly proportional to the square root of the tension, T, and inversely proportional to the length, L. What is the effect on the frequency if the length is doubled and the tension is quadrupled?

$$f = \frac{K\sqrt{T}}{L}$$

$$f = \frac{K\sqrt{4T}}{2L}$$

$$= \frac{K \cdot 2\sqrt{T}}{2L}$$

$$f = \frac{K\sqrt{T}}{L}$$

Nothing changes

COMBINED VARIATION

When a number varies both directly and indirectly

You try!

1)   2)

3)

SUMMARY: