Write your questions and thoughts here!

$y = 2^x$

7.1 – Exponential Functions

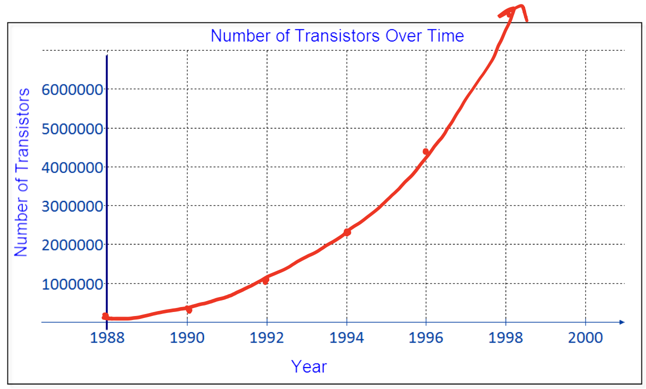
Name:____________________

Moore's Law states that processor speeds, or overall processing power for computers will double about every 18 months. Likewise, the cost to produce a comparable computer will be cut in half.

What does this mean? To simplify the mathematics, let's assume it takes 2 yrs to double/half instead of 18 months.

Example 1: In 1988, the number of transistors in the Intel 386 SX microprocessor was 275,000. What was the approximate transistor count of the Pentium II Intel microprocessor in 1998?

Year	*Transistors*
1988	275,000
1990	550,000
1992	1,110,000
1994	2,200,000
1996	4,400,000
1998	8,800,00

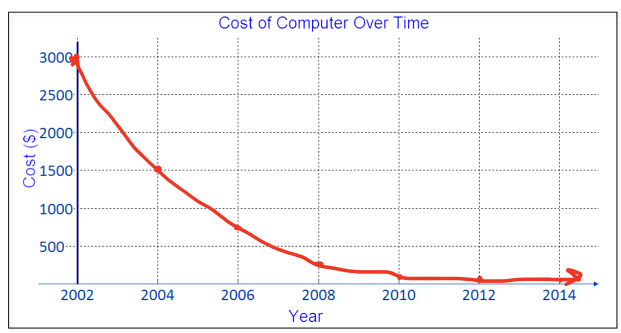


This is called exponential GROWTH.

(The actual number of transistors in in a Pentium II chip in 1998 was 7,500,000!!)

Example 2: A personal computer that cost $3,000 in 2002 would cost about how much now?

Year	*Cost ($)*
2002	3000
2004	1500
2006	750
2008	375
2010	187.50
2012	93.75
2014	46.88



This is called Exponential DECAY.

An ***Exponential Function*** is of the form:

$$y = a(b)^x$$

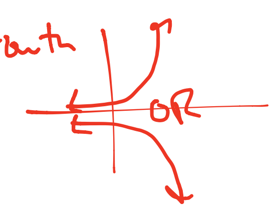
Initial Value

Growth/decay factor

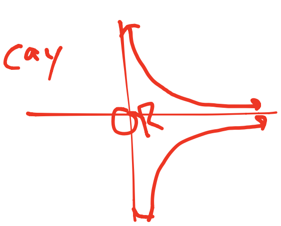
$b > 1$ $0 < b < 1$

Condition 1: $a \neq 0$
Condition 2: The base (b) is a positive number other than 1.

growth



Decay



Write your questions and thoughts here!

7.1 – Exponential Functions

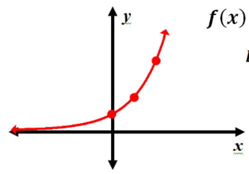
EXPONENTIAL GROWTH FUNCTIONS:

If $b > 1$, then the graph will "grow" away from the asymptote as you move left to right.

The x-axis ($y = 0$) is an **ASYMPTOTE** of the graph. An asymptote is an imaginary line that a graph approaches more and more **closely**.

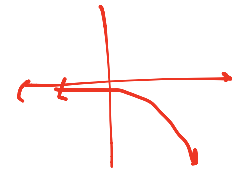
$f(x) = a(b)^x$

$b > 1$



Domain: all real numbers
Range: $y > 0$

Also Possible:

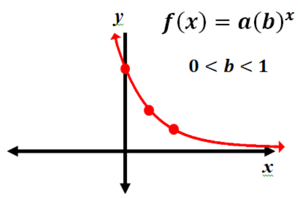


EXPONENTIAL DECAY FUNCTIONS:

If $0 < b < 1$, then the graph will "decay" toward the asymptote as you move left to right.

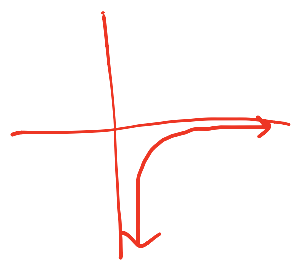
$f(x) = a(b)^x$

$0 < b < 1$



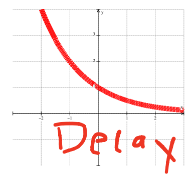
Domain: all real numbers
Range: $y > 0$

Also Possible:

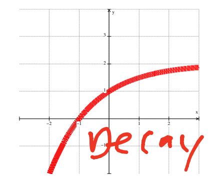


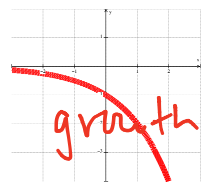
You decide! Growth, decay or neither?

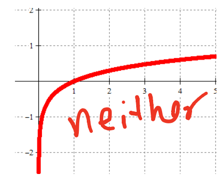
$y = \frac{1}{2} \cdot 2^x$ Growth



$y = 2 \cdot \frac{1}{2}^x$ Decay

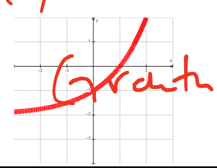






Decay $y = -2^{-x} = -(\frac{1}{2})^x$

$y = 0.4\left(\frac{7}{10}\right)^{-x}$ Growth



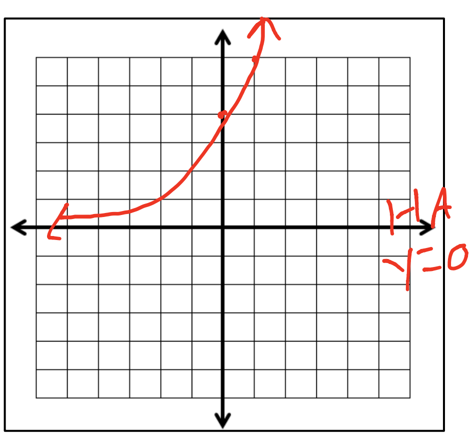
$f(x) = \frac{1}{2} \cdot x^2$ neither

Graphing Shortcut:

For exponential functions that are in the form $y = a(b)^x$, the graph will go through: $(0, a)$, $(1, ab)$, $(2, ab^2)$ and have an asymptote on the x-axis

Cool website → https://www.desmos.com/calculator

3. Graph $y = 3 \cdot \left(\frac{1}{2}\right)^{-x}$ $= 3 \cdot (2)^x$



x	y
-2	3/4
-1	$\frac{3}{2}$
0	3
1	6
2	12

Growth

Domain: $\mathbb{R}$ Range: $(0, \infty)$

Write your questions and thoughts here!

7.1 – Exponential Functions

Remember these important formulas:

COMPOUND INTEREST

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

t = **time** (years) P = **principal** (initial investment)
r = **annual interest rate**
n = the number of times the interest is **compounded** (paid) per year.

GROWTH AND DECAY MODELS

$$\boldsymbol{y = a(1 \pm r)^t}$$

t = **time** a = **initial value**
r = **rate of change**
+ = increase - = decrease

CONTINUOUSLY COMPOUNDED INTEREST

$$A = Pe^{rt}$$

t = **time** (years) P = **principal** (initial investment)
r = **annual interest rate**
A = **amount in the account**

4. You deposit $3,000 in an account that pays 3.28% annual interest. How much would you have if the interest was compounded daily for five years? Continuously?

P $r=.0328$ $n=365$ t

$A=P\left(1+\frac{r}{n}\right)^{nt}$

$=3000\left(1+\frac{.0328}{365}\right)^{(365)(5)}$

$A=\$3534.62$

$A=Pe^{rt}$

$=3000e^{0.0328(5)}$

$A=\$3534.64$

5. The relative growth rate for bacteria from Sully's tongue is 80% per hour after eating a Turkish Pizza. Sully swabs his mouth and starts a culture that, 4 hours later, shows approximately 50,000 bacteria. How many bacteria did Sully start the culture with?

$r=+.80$ t y

$y=a(1\pm r)^t$

$50{,}000=a(1+.80)^4$

$50{,}000=a(1.80)^4$

$\frac{50{,}000}{1.80^4}=a$

$a \approx 4763$ bacteria

Remember your rules when solving equations with exponents!!

You try: Solve for r.

6. $\frac{3^{x^2}}{27} = 9^x$

$\frac{3^{x^2}}{3^3}=(3^2)^x$

$3^{x^2-3}=3^{2x}$

$x^2-3=2x$

$x^2-2x-3=0$

$(x-3)(x+1)=0$

$x-3=0$, $x+1=0$

$x=3$, $x=-1$

7. $64^{-3w}\cdot\frac{1}{64} = 4^2$

$(4^3)^{-3w}\cdot 4^{-3}=4^2$

$4^{-9w}\cdot 4^{-3}=4^2$

$4^{-9w-3}=4^2$

$-9w-3=2$

$-9w=5$

$w=-5/9$

8. $16^{2r+3}\cdot\left(\frac{1}{16}\right)^{2r} = 4$

$(4^2)^{2r+3}\,(4^{-2})^{2r}=4^1$

$4^{4r+6}\cdot 4^{-4r}=4^1$

$4^6=4^1$

$6\neq 1$

No solution

$\sum \mathbb{R}e$