7.2 – Logarithmic Functions

Name:_____

Write your questions and thoughts here!

Definition of Logarithm:

$y = \log_b x$ if and only if $b^y = x$

$b > 0, b \neq 1, x > 0$

$2^3 = 8 \iff \log_2 8 = 3$

How does Mr. Bean say $\log_b x$?

log base b of x

How does Mr. Kelly say $\log_b x$ in his head?

b to what exponent is x?

These functions are inverse functions!

Recall: An exponential function is of the form $y = b^x$

A Logarithmic function is of the form $x = b^y$

Rewrite in exponential form:

1. $\log_5 125 = 3$

$5^3 = 125$

Rewrite in logarithmic form:

2. $\sqrt[3]{\frac{1}{8}} = \frac{1}{2}$

$\left(\frac{1}{8}\right)^{\frac{1}{3}} = \frac{1}{2}$

$\log_{\frac{1}{8}} \frac{1}{2} = \frac{1}{3}$

Evaluate the following logarithms (without a calculator):

3. $\log_2 8 = x$

$2^x = 8$

$2^x = 2^3$

$x = 3$

4. $\log_8 \frac{1}{64} = \log_8 8^{-2} = -2$

5. $\log_8 1 = \log_8 8^0 = 0$

log ID

1) $\log_a a^x = x$

2) $a^{\log_a x} = x$

Property of Equality

If $\log_a x = \log_a y$

then $x = y$

PROPERTIES OF LOGARITHMS

Product Property:	Quotient Property:	Power Property:
$\log_b xy = \log_b x + \log_b y$	$\log_b \frac{x}{y} = \log_b x - \log_b y$	$\log_b x^y = y \log_b x$

EXPANDING AND CONDENSING LOGRITHMS:

Expand:

6. $\log_4 3x^5y$

$= \log_4 3 + \log_4 x^5 + \log_4 y$

$= \log_4 3 + 5\log_4 x + \log_4 y$

7. $\log_3 \left(\frac{p}{q^4}\right)^5 = \log_3 \left(\frac{p^5}{q^{20}}\right)$

$= \log_3 p^5 - \log_3 q^{20}$

$= 5\log_3 p - 20\log_3 q$

Condense:

8. $2(\log_4 8 - \log_4 5) + 0.5\log_4 25$

$= 2\log_4 8 - 2\log_4 5 + \frac{1}{2}\log_4 25$

$= \log_4 64 - \log_4 25 + \log_4 5$

$= 3 + \log_4 5 - \log_4 25$

$= 3 + \log_4 \frac{5}{25} = 3 + \log_4 \frac{1}{5}$

CHANGE-OF-BASE:

$\log_c a = \frac{\log_b a}{\log_b c}$

9. Evaluate $\log_{12} 5$

$\frac{\log 5}{\log 12} \approx .6477$

10. Evaluate $\log_2 20$

$\frac{\log 20}{\log 2} = 4.3219$

Method:

Solve for x:	Bean: (take log both sides)	Brust: (cancel by using log of base)	Sully: JUST GRAPH IT, BABY!
11. $41 = 7^x$	$\log 41 = \log 7^x$ $\log 41 = x \log 7$ $\frac{\log 41}{\log 7} = x$	$\log_7 41 = \log_7 7^x$ $\log_7 41 = x$	$y_1 = 41$ $y_2 = 7^x$ 1.908
12. $0.25 = 2^x$	$\log \frac{1}{4} = \log 2^x$ $\log 2^{-2} = x \log 2$ $-2 \log 2 = x \log 2$ $-2 = x$	$\log_2 \frac{1}{4} = \log_2 2^x$ $\log_2 2^{-2} = x$ $-2 = x$	-2

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Other Reminders:

$\log_b b = 1$ $\quad \log_b 1 = 0$ $\quad \log_{10} x$ is called: Common log $= \log x$

$\log_b b^x = x$ $\quad b^{\log_b x} = x$ $\quad \log_e x$ is called: natural log $= \ln x$

Solve for the unknown variable.

14. $\log x = 3.0876$

$10^{\log_{10} x} = 10^{3.0876}$

$x = 10^{3.0876}$

$x \approx 1223.4888$

15. $\ln x = -0.9128$

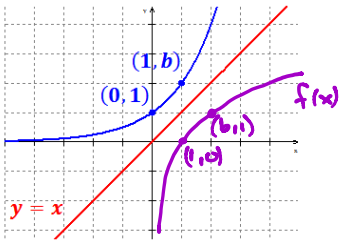
$e^{\ln x} = e^{-0.9128}$

$x = e^{-0.9128}$

$x \approx 0.4014$

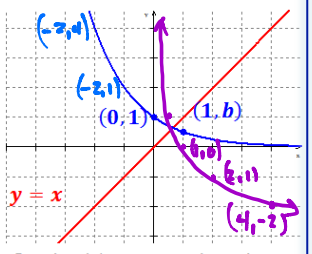
GRAPHING LOGARITHMIC FUNCTIONS:

$g(x) = b^x$ where $b > 1$



Graph $f(x) = \log_b x$ above by reflecting $g(x)$ across the line $y = x$.

$g(x) = b^x$ where $0 < b < 1$



Graph $f(x) = \log_b x$ above by reflecting $g(x)$ across the line $y = x$.

Use the properties of logarithms to find the inverse of the given function. *(Hint! Switch x and y and solve for y!)*

16. $f(x) = 4^x$

$y = 4^x$

$x = 4^y$

$\log_4 x = y$

$\log_4 x = f^{-1}(x)$

17. $f(x) = 3^{2x}$

$y = 3^{2x}$

$x = (3^2)^y$

$x = 9^y$

$\log x = \log 9^y$

$\log x = y \log 9$

$\frac{\log x}{\log 9} = y$

$\frac{\log x}{\log 9} = f^{-1}(x)$

18. $f(x) = 5^{\left(\frac{1}{2}x\right)}$

$y = (5^{\frac{1}{2}})^x$

$x = \sqrt{5}^y$

$\log x = \log \sqrt{5}^y$

$\log x = y \log \sqrt{5}$

$\frac{\log x}{\log \sqrt{5}} = y$

$\frac{\log x}{\log \sqrt{5}} = f^{-1}(x)$

19. $f(x) = \log x + 1$

$y = \log x + 1$

$x = \log y + 1$

$x - 1 = \log y$ (log form)

$10^{x-1} = y$ (exponential form)

20. $\ln y = 2 \ln x$

$\ln x = 2 \ln y$

$\frac{1}{2} \ln x = \ln y$

$\ln x^{\frac{1}{2}} = \ln y$

$\sqrt{x} = y$

$\sum \mathbb{R} e$