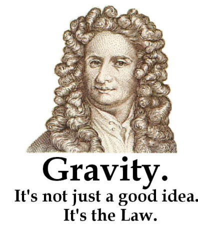
7.3 – Exp & Log Problem Solving

Write your questions and thoughts here!

Useful stuff to know:

Newton's Law of Cooling:

$$T(t) = T_s + (T_0 - T_s)e^{-kt}$$



- $T(t)$ is the temperature of the object after time t.
- T_s is the temperature of the surrounding environment.
- T_0 is the initial temperature of the object (at time $t = 0$).
- k is a constant that changes depending on the material properties of the object.
- t is the amount of time (in minutes) that has passed since the object began cooling.

Decibel Levels:

$$D(I) = 10\log\left(\frac{I}{I_o}\right)$$

$D(I)$ is the decibel level (loudness) as a function of I.
I is the intensity of the sound (watts per square meter).
$I_o = 10^{-12}$ and represents the intensity of the quietest sound a human can hear.

chter Scale of Earthquake Energy

h level is 10 time stronger than the previous level

	Description	Occurrence	In Population	Movement
1	Small	Daily	Every minute	Small
2	Small	Daily	Every hour	Small
3	Small	Daily	Every day	Small
4	Small	Daily	Every week	Moderate sudden
5	Moderate	Monthly	Every 10 years	Strong Sudden
6	Moderate	Monthly	Every 30 years	Strong Sudden
7	Major	Monthly	Every 50 years	Severe Sudden
8	Great	Yearly	Every 100 years	Very Severe
9	Great	Yearly	Every 300 years	Very Severe
10	Super	Rarely	Every 1.000 years	Extreme

Earthquake Intensity (Richter Scale):

$$M = \frac{2}{3}\log\frac{E}{E_o}$$

M is the magnitude of the earthquake
E = the energy released by the earthquake (joules)
$E_o = 10^{4.40}$ joules, the energy released by a small reference earthquake

Rocket Science

$$v = c\ ln\frac{W_t}{W_b}$$

v is the velocity of a rocket at fuel burnout (when it runs out of fuel)
W_t = the takeoff weight (fuel, structure and payload)
W_b = burnout weight (only structure and payload) c = exhaust velocity of the rocket

Half-Life Decay

$$A = A_o\left(\frac{1}{2}\right)^{t/h} \text{ or } A_o(2)^{-t/h}$$

A is the amount at time t
A_o = the amount at time = 0
h = half-life

Carbon-14 Dating

$$A = A_o e^{-0.000124t}$$

A is the amount after t years
A_o = the amount at time = 0
t = time

The amount of carbon-14 remains constant as long as a plant or animal is alive. Once dead, the organism ceases to take in carbon-14, and the remaining carbon-14 starts to decay. This can be used to date artifacts that are super old, like Bean's jokes.

COMPOUND INTEREST

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

t = time (years) P = principal (initial investment)
r = annual interest rate
n = the number of times the interest is compounded (paid) per year.

GROWTH AND DECAY MODELS

$$\boldsymbol{y = a(1 \pm r)^t}$$

t = time a = initial value
r = rate of change
+ = increase - = decrease

CONTINUOUSLY COMPOUNDED INTEREST

$$A = Pe^{rt}$$

t = time (years) P = principal (initial investment)
r = annual interest rate
A = amount in the account

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10. Bean visits *"The New York Metzgerei"* for Sully's Saturday special: Grilled Brust-Wurst! We know the Brust-Wurst is cooked at 325°F and the room temperature is 72°F. From the moment it comes off the grill, we wait two minutes and then measure the temperature to be 205°F. How long after taking the Brust-Wurst off of the grill should Bean eat it? (120°F is a safe temperature to eat food.)

FIND K

$T(t) = 205°$
$T_s = 72°$
$T_o = 325°$
$k =$
$t = 2$

$T(t) = T_s + (T_o - T_s)e^{-kt}$
$205 = 72 + (325-72)e^{-k(2)}$
$133 = 253e^{-2k}$
$\frac{133}{253} = e^{-2k}$
$\ln\frac{133}{253} = -2k$
$k = -\frac{1}{2}\ln\frac{133}{253}$
$k \approx .3215$

$T(t) = 120°$
$T_s = 72°$
$T_o = 325°$
$k = .3215$
$t =$

$T(t) = T_s + (T_o - T_s)e^{-kt}$
$120 = 72 + (325-72)e^{-.3215t}$
$48 = 253e^{-.3215t}$
$\frac{48}{253} = e^{-.3215t}$
$\ln\frac{48}{253} = -.3215t$

$t = \frac{\ln 48}{253}\left(\frac{1}{-.3215}\right)$
$t \approx 5.1698$ minutes

11. The atomic bomb dropped on Nagasaki in 1945 released about 1.34 x 10^{14} joules of energy. What would be the magnitude of an equivalent earthquake that released that much energy?

$M =$
$E = 1.34 \; 10^{14}$
(From formula) $E_0 = 10^{4.40}$

$M = \frac{2}{3}\log\frac{E}{E_0}$
$M = \frac{2}{3}\log\frac{1.34 \cdot 10^{14}}{10^{4.40}}$
$M \approx 6.4847$

12. How many years will it take \$5,000 to amount to \$8,000 if it is invested at an annual rate of 9% compounded continuously?

$A = 8000$
$P = 5000$
$r = .09$

$A = Pe^{rt}$
$8000 = 5000e^{.09t}$
$\frac{8}{5} = e^{.09t}$
$\ln\frac{8}{5} = .09t$
$\frac{100}{9}\ln\frac{8}{5} = t$
$5.2223 \approx t$

13. In 2004, archaeologist Al Goodyear discovered a site in South Carolina that contains evidence of the earliest human settlement in North America. Carbon dating of burned plant material indicated 0.2% of the amount of carbon-14 in a sample. How old was the sample?

$A = .002$
$A_o = 1$
$t =$

$A = A_0 e^{-.000124t}$
$.002 = 1 \cdot e^{-.000124t}$
$\ln .002 = -.000124t$
$\frac{\ln .002}{-.000124} = t$
$50{,}117.8073 \approx t$

Now summarize what you learned!