Application 7.3

1. $\ln x^6 = 12$

$6\ln x = 12$

$\ln x = 2$

$e^2 = x$

$7.389 \approx x$

2. $\log(t-4) = -1$

$10^{-1} = t-4$

$\frac{1}{10} = t-4$

$.1 = t-4$

$4.1 = t$

3. A hard-boiled egg at temperature 96^oC is placed in 16^oC water to cool. Four minutes later the temperature of the egg is 45^oC. Use Newton's Law of Cooling to find k.

T_o T_s $t=4$ $T(t)$

$T(t) = T_s + (T_o - T_s)e^{-kt}$

$45 = 16 + (96-16)e^{-k(4)}$

$29 = 80e^{-4k}$

$\frac{29}{80} = e^{-4k}$

$\ln\frac{29}{80} = -4k$

$-\frac{1}{4}\ln\frac{29}{80} = k$

$.2537 \approx k$

Now, use k to find the *time* when $T = 20^0C$.

$20 = 16 + (96-16)e^{-.2537t}$

$4 = 80e^{-.2537t}$

$\frac{1}{20} = e^{-.2537t}$

$\ln\frac{1}{20} = -.2537t$

$\frac{\ln\frac{1}{20}}{-.2537} = t$

$11.8 \approx t$

It would take about 11.8 minutes.

4. There were 12 earthquakes recorded word-wide in 2008 with a magnitude at least 7.0. $M = \frac{2}{3}\log\frac{E}{10^{4.4}}$

a. How much energy is released by a magnitude 7.0 earthquake?

$M = \frac{2}{3}\log\frac{E}{10^{4.4}}$

$7 = \frac{2}{3}\log\frac{E}{10^{4.4}}$

$\frac{21}{2} = \log\frac{E}{10^{4.4}}$

$10.5 = \log\frac{E}{10^{4.4}}$

$10^{10.5} = \frac{E}{10^{4.4}}$

$10^{4.4}\cdot 10^{10.5} = E$

$10^{14.9} = E$

$E \approx 7.94 \cdot 10^{14}$ joules

b. The total average daily consumption of energy for the entire United States in 2006 was 2.88 x 10^{14} joules. How many days could the energy released by a magnitude 7.0 earthquake power the United States?

$\frac{\text{Earthquake Energy}}{\text{total daily consumpt}} = \frac{7.94\cdot 10^{14}}{2.88\cdot 10^{14}} = \frac{7.94}{2.88} \approx 2.757$ days

5. A new liquid fueled rocket has a weight ratio of $\frac{W_t}{W_b} = 6.2$ and an exhaust velocity of c = 5.2 km/sec. What is the velocity at burnout?

$V = c\ln\frac{W_t}{W_b}$

$V = (5.2\frac{km}{sec})\ln(6.2)$

$V \approx 9.488$ km/s

6. Generally, an earthquake requires a magnitude of over 5.6 on the Richter scale to inflict serious damage. How many more times powerful was this earthquake than the one in Columbia in 1906, which registered an 8.6?

$M = \frac{2}{3}\log_{10}\frac{E}{E_o}$

Thus $\frac{10^{8.6}}{10^{5.6}} = 10^3 = 1000$ times greater

7.3 Exponential and Logarithmic Problem Solving Practice

7. How many times larger is the ~~sound intensity~~ Decibal level from Sully's classroom before Sully Law is enacted (8.5×10^{-4}) compared to when after Sully law is enacted (5.2×10^{-10}).

Before
$$\boxed{\begin{aligned} D(I) &= 10\log\frac{I}{I_0} \\ &= 10\log\frac{8.5\cdot 10^{-4}}{10^{-12}} \\ D(I) &= 10(\log 8.5\times 10^{8}) \end{aligned}}$$

After
$$\boxed{\begin{aligned} D(I) &= 10\log\frac{I}{10^{-12}} \\ &= 10\log\frac{5.2\times 10^{-10}}{10^{-12}} \\ D(I) &= 10\log(5.2\times 10^{2}) \end{aligned}}$$

$$\frac{\text{Before}}{\text{After}} = \frac{\cancel{10}\log(8.5\times 10^{8})}{\cancel{10}\log(5.2\times 10^{2})}$$
$= 3.288$ times louder

8. The earliest mechanical clocks appeared around 1350 in Europe, and would gain or lose an average of 30 minutes per day. After that, accuracy doubled every 30 years. Find the predicted accuracy of clocks in the year:

$$\text{Error} = 30\left(\tfrac{1}{2}\right)^{\frac{t}{30}}$$

a. 1700

$$\boxed{\begin{aligned} t &= 1700-1350 \\ t &= 350 \end{aligned}}$$

$$= 30\left(\tfrac{1}{2}\right)^{\frac{350}{30}}$$

Error = .009 minutes/day

b. 2000

$$\boxed{\begin{aligned} t &= 2000-1350 \\ t &= 650 \end{aligned}}$$

$$\text{Error} = 30\left(\tfrac{1}{2}\right)^{\frac{t}{30}}$$
$$= 30\left(\tfrac{1}{2}\right)^{\frac{650}{30}}$$

Error $\approx 9.01\times 10^{-6}$ m/day

9. In 2003, Japanese scientists announced an effort to bring back the long-extinct Woolly-Mammoth using cloning techniques. They found a well preserved specimen in Siberia. Nearby plant specimens were found to have 28.9% of the amount of carbon-14 in a living sample. How old is the specimen found in Siberia?

$$A = A_0 e^{-0.000124t}$$
$$.289 = 1e^{-0.000124t}$$
$$\ln .289 = -.000124t$$
$$\frac{\ln .289}{-.000124} = t$$

$t \approx 10{,}011$ years old

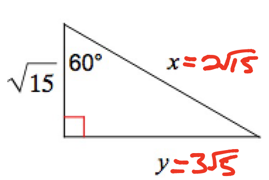
10. At what rate compounded continuously will \$1000 have to be invested to amount to \$2500 in 10 years?

$$A = Pe^{rt}$$
$$2500 = 1000e^{r(10)}$$
$$2.5 = e^{10r}$$
$$\ln 2.5 = 10r$$
$$\frac{\ln 2.5}{10} = r$$
$$.0916 \approx r$$

9.16%

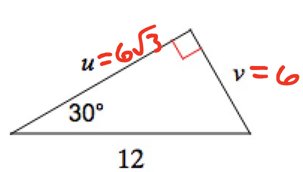
Quick Review. Find the missing variable.

11.

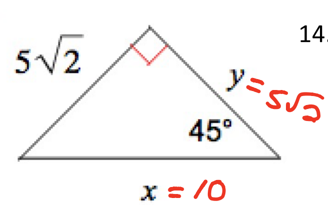


$$\sqrt{15}\cdot\sqrt{3} = \sqrt{3\cdot 3\cdot 5} = 3\sqrt{5}$$

12.



13.



14.