

7.3 Corrective Assignment

Solve, rounding to three significant digits.

1. $8^{-x} = 0.654$
 $\log_8 8^{-x} = \log_8 0.654$
 $-x = \frac{\log 0.654}{\log 8}$
 $x = \frac{-\log 0.654}{\log 8}$
 $x \approx 0.204$

2. $10^{x-4} = 92$
 $\log 10^{x-4} = \log 92$
 $x-4 = \log 92$
 $x = 4 + \log 92$
 $x \approx 5.964$

3. $e^x = 8.88$
 $\ln e^x = \ln 8.88$
 $x = \ln 8.88$
 $x \approx 2.184$

4. $15e^{2x-1} = 272$
 $e^{2x-1} = \frac{272}{15}$
 $\ln e^{2x-1} = \ln \frac{272}{15}$
 $2x-1 = \ln \frac{272}{15}$
 $2x = 1 + \ln \frac{272}{15}$
 $x = \frac{1}{2} + \frac{1}{2} \ln \frac{272}{15}$
 $x \approx 1.949$

5. $\log_8 x = 2$
 $8^2 = x$
 $64 = x$

6. $e^{0.045x} + 105 = 240$
 $e^{0.045x} = 135$
 $\ln e^{0.045x} = \ln 135$
 $0.045x = \ln 135$
 $x = \frac{\ln 135}{0.045}$
 $x \approx 109.066$

7. $\log x + \log 10 = 2$
 $\log x + 1 = 2$
 $\log x = 1$
 $10^{\log x} = 10^1$
 $x = 10$

8. $\log(x+4) + \log(x) = 1$
 $\log(x^2+4x) = 1$
 $10^{\log(x^2+4x)} = 10^1$
 $x^2+4x = 10$
 $x^2+4x-10 = 0$
 $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
 $x = \frac{-4 \pm \sqrt{16+40}}{2}$
 $x = \frac{-4 \pm \sqrt{56}}{2}$
 $x = \frac{-4 \pm 2\sqrt{14}}{2}$
 $x = -2 \pm \sqrt{14}$
 $x \approx 1.742$
check for extraneous solutions!
 $x \neq -2 - \sqrt{14}$ (Neg Argument)

9. $\log(15-2x) = \log(3x-20)$
 $15-2x = 3x-20$
 $15 = 5x-20$
 $35 = 5x$
 $7 = x$

10. $5 - \log(x+2) = 7$
 $-\log(x+2) = 2$
 $\log(x+2) = -2$
 $10^{-2} = x+2$
 $\frac{1}{100} = x+2$
 $-2 + \frac{1}{100} = x$
 $-1.990 = x$

11. $5^{x+1} = 17$
 $\log_5 5^{x+1} = \log_5 17$
 $x+1 = \log_5 17$
 $x = -1 + \frac{\log 17}{\log 5}$
 $x \approx 0.760$

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12. A Brust-Wurst is boiled at 98°C and then placed in 16°C water to cool. Five minutes later the temperature of the Brust-Wurst is 35°C. Use Newton's Law of Cooling to find k.

$$T(t) = T_s + (T_0 - T_s)e^{-kt}$$

$$35 = 16 + (98 - 16)e^{-k(5)}$$

$$35 = 16 + 82e^{-5k}$$

$$19 = 82e^{-5k}$$

$$\frac{19}{82} = e^{-5k}$$

$$\ln \frac{19}{82} = \ln e^{-5k}$$

$$\ln \frac{19}{82} = -5k$$

$$-\frac{1}{5} \ln \frac{19}{82} = k$$

$$.2925 \approx k$$

$$T(t) = T_s + (T_0 - T_s)e^{-kt}$$

$$22 = 16 + (98 - 16)e^{-(.2925)t}$$

$$22 = 16 + 82e^{-.2925t}$$

$$6 = 82e^{-.2925t}$$

$$\frac{6}{82} = e^{-.2925t}$$

$$\ln \frac{6}{82} = \ln e^{-.2925t}$$

$$\ln \frac{6}{82} = -.2925t$$

$$\frac{\ln \frac{6}{82}}{-.2925} = t$$

$$8.94 \approx t$$

Now, use k to find the time when T = 22°C.

13. How many years will it take for carbon-14 to diminish to 3% of its original amount after the death of a plant or animal?

$$A = A_0 e^{-0.000124t}$$

$$.03 = 1 \cdot e^{-0.000124t}$$

$$\ln .03 = \ln e^{-0.000124t}$$

$$\ln .03 = -0.000124t$$

$$\frac{\ln .03}{-0.000124} = t$$

$$28,278.693 \text{ years} \approx t$$

$$A = .03$$

$$A_0 = 100\% = 1$$

14. At what rate compounded quarterly will \$1000 have to be invested to amount to \$2500 in 10 years?

$$A = P(1 + \frac{r}{n})^{nt}$$

$$2500 = 1000(1 + \frac{r}{4})^{4(10)}$$

$$2.5 = (1 + \frac{r}{4})^{40}$$

$$\pm \sqrt[40]{2.5} = 1 + \frac{r}{4}$$

$$-1 + \sqrt[40]{2.5} = \frac{r}{4}$$

$$-4 + 4\sqrt[40]{2.5} = r$$

$$r \approx .0927$$

$$r \approx 9.27\%$$

15. A medical researcher is testing a radioactive isotope for use in a new imaging process. She finds that an original sample of 5 grams decays to 1 gram in 6 hours. Find the half-life of the sample to three significant digits. (Use half-life formula.)

$$A = A_0(2)^{-t/h}$$

$$1 = 5(2)^{-6/h}$$

$$\frac{1}{5} = (2)^{-6/h}$$

$$\log_2 \frac{1}{5} = \log_2 (2)^{-6/h}$$

$$\log_2 \frac{1}{5} = -6/h$$

$$h \cdot \log_2 \frac{1}{5} = -6$$

$$h = \frac{-6}{\log_2 \frac{1}{5}}$$

$$h = \frac{-6}{\frac{\log \frac{1}{5}}{\log 2}}$$

$$h = \frac{-6 \log 2}{\log \frac{1}{5}}$$

$$h = 2.584 \text{ hours}$$

ANSWERS:

1. x = 0.204
2. x = 5.964
3. x = 2.184
4. x = 1.949
5. x = 64.000
6. x = 109.006
7. x = 10.000
8. Both x = 0 and x = -4 are extraneous. Therefore, no solution.
9. x = 7.000
10. x = -1.990
11. x = .386
12. k = 0.292456; time = 8.941 min
13. 28,278.698 years
14. r = 9.269%
15. 0.035068 hours or 2.1 min