PRACTICE

Perform each conversion.

1. 45 yards = 1620 inches

$45\text{ yd} \cdot \frac{3\text{ ft}}{1\text{ yd}} \cdot \frac{12\text{ in}}{1\text{ ft}}$

2. 1.5 km = 150,000 cm

$1.5\text{ km} \cdot \frac{1000\text{ m}}{1\text{ km}} \cdot \frac{100\text{ cm}}{1\text{ m}}$

3. 28 m/sec = 100.8 km/hr

$\frac{28\text{ m}}{1\text{ sec}} \cdot \frac{1\text{ km}}{1000\text{ m}} \cdot \frac{60\text{ sec}}{1\text{ min}} \cdot \frac{60\text{ min}}{1\text{ hr}}$

$= \frac{100800\text{ km}}{1000\text{ hr}}$

$= 100.8$

4. 30 rev/min = 60π rad/min

$\frac{30\text{ rev}}{1\text{ min}} \cdot \frac{2\pi\text{ RAD}}{1\text{ rev}}$

5. 120 rev/hr = 2 rev/min

$\frac{120\text{ Rev}}{1\text{ hr}} \cdot \frac{1\text{ hr}}{60\text{ min}}$

6. 30 mi/hr = 44 ft/sec

$\frac{30\text{ mi.}}{1\text{ hr}} \cdot \frac{5280\text{ ft}}{1\text{ mi.}} \cdot \frac{1\text{ hr}}{60\text{ min}} \cdot \frac{1\text{ min}}{60\text{ sec}}$

$\frac{158400\text{ ft}}{3600\text{ sec}} = 44\text{ ft/sec}$

A windmill for generating electricity has a blade that is 30 feet long. Depending on the wind, it rotates at various velocities. In each case, find the angular velocity in rad/sec for the tip of the blade.

7. 500 rev/sec

$\frac{500\text{ Rev}}{1\text{ Sec}} \cdot \frac{2\pi\text{ RAD}}{1\text{ Rev}}$

$= 1000\pi\text{ RAD/sec}$

8. 11,000 rev/hr

$\frac{11{,}000\text{ Rev}}{1\text{ hr}} \cdot \frac{2\pi\text{ RAD}}{1\text{ Rev}} \cdot \frac{1\text{ hr}}{60\text{ min}} \cdot \frac{1\text{ min}}{60\text{ sec}}$

$= \frac{22{,}000\pi\text{ RAD}}{3600\text{ sec}}$

$= \frac{220\pi}{36}\text{ RAD/sec}$

$= \frac{55}{9}\pi\text{ RAD/sec}$

9. 50,000 rev/day

$\frac{50{,}000\text{ Rev}}{1\text{ day}} \cdot \frac{2\pi\text{ RAD}}{1\text{ Rev}} \cdot \frac{1\text{ day}}{24\text{ hr}} \cdot \frac{1\text{ hr}}{60\text{ min}} \cdot \frac{1\text{ min}}{60\text{ sec}}$

$= \frac{100{,}000\pi}{86{,}400}\text{ RAD/sec}$

$= \frac{1000\pi}{864}$

$= \frac{125\pi}{108}\text{ RAD/sec}$

A common speed for an electric motor is 3450 revolutions per minute. Saw blades of various diameters can be attached. Determine the linear velocity in mi/hr for a point on the edge of a blade given the diameter.

10. 6 in = d

Circumference

$\frac{3450\text{ Rev}}{1\text{ min}} \cdot \frac{6\pi\text{ in}}{1\text{ Rev}} \cdot \frac{1\text{ ft}}{12\text{ in}} \cdot \frac{1\text{ mile}}{5280\text{ ft}}$

$\cdot \frac{60\text{ min}}{1\text{ hr}} = \frac{1{,}242{,}000\pi\text{ mile}}{63{,}360\text{ hr}}$

$\approx 61.582\text{ mile/hr}$

11. 1.2 feet = d

$\frac{3450\text{ Rev}}{1\text{ min}} \cdot \frac{1.2\pi\text{ ft}}{1\text{ Rev}} \cdot \frac{1\text{ mile}}{5280\text{ ft}} \cdot \frac{60\text{ min}}{1\text{ hr}}$

$= 147.798\text{ mi/hr}$

12. 1 yard = d

$\frac{3450\text{ Rev}}{1\text{ min}} \cdot \frac{1\pi\text{ yd}}{1\text{ Rev}} \cdot \frac{3\text{ ft}}{1\text{ yd}} \cdot \frac{1\text{ mile}}{5280\text{ ft}}$

$\frac{60\text{ min}}{1\text{ hr}} \approx 3609.494\text{ mi/hr}$

13. A circular blade with a 12-inch diameter spins at a rate of 1800 rpm (revolutions per minute).

a. What is the blade's angular velocity in radians per minute?

$\frac{1800 \text{ rev}}{1 \text{ min}} \cdot \frac{2\pi \text{ RAD}}{1 \text{ Rev}} = 3600\pi \text{ RAD/min}$

b. Find the linear velocity (in inches per minute) of one of the teeth on the edge of the blade.

$\frac{1800 \text{ rev}}{1 \text{ min}} \cdot \frac{12\pi}{1 \text{ Rev}}$ (Circumf.) $= 21{,}600\pi \text{ in/min} \approx 67858.401 \text{ in/min}$

c. Convert the linear velocity into feet per second.

$\frac{21{,}600\pi \text{ in}}{1 \text{ min}} \cdot \frac{1 \text{ ft}}{12 \text{ in}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = \frac{21{,}600\pi \text{ ft}}{720 \text{ sec}} \approx 94.248 \text{ ft/sec}$

14. Vinyl record albums are 11 inches in diameter and spin at a rate of 33 rpm.

a. What is a record's angular velocity in radians per minute?

$\frac{33 \text{ rev}}{1 \text{ min}} \cdot \frac{2\pi \text{ RAD}}{1 \text{ Rev}} = 66\pi \text{ RAD/min}$

b. How fast (in inches per minute) would a record move under a needle placed on the records edge?

$\frac{33 \text{ Rev}}{1 \text{ min}} \cdot \frac{11\pi \text{ in}}{1 \text{ Rev}}$ (Circumference) $= 363\pi \text{ in/min} \approx 1140.398 \text{ in/min}$

c. Convert this linear velocity to feet per second.

$\frac{363\pi \text{ in}}{1 \text{ min}} \cdot \frac{1 \text{ ft}}{12 \text{ in}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = \frac{363\pi \text{ ft/sec}}{720} \approx 1.584 \text{ ft/sec}$

15. With his arms fully extended, a baseball player swings a bat. Using his shoulder as the center of rotation, the bat moves through 120° in only 0.2 seconds.

a. What is the angular velocity of the batter's swing in radians per second?

$\frac{120^\circ}{0.2 \text{ sec}} \cdot \frac{\pi \text{ RAD}}{180^\circ} = \frac{12\pi}{3.6} \text{ RAD/sec} = \frac{120\pi \text{ RAD}}{36 \text{ sec}} = \frac{10\pi}{3} \text{ RAD/sec}$

b. As he swings the bat, the player hits a baseball. Suppose the ball leaves the bat at a distance of 40 inches from the player's shoulder. How fast (in miles per hour) would the ball be moving?

$r = 40 \text{ in}$

$\frac{10\pi \text{ RAD}}{3 \text{ sec}} \cdot \frac{2\pi(40 \text{ in})}{2\pi \text{ RAD}}$ (Circumference) $\cdot \frac{1 \text{ ft}}{12 \text{ in}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} = \frac{1{,}360{,}000\pi \text{ mi}}{190{,}080 \text{ hr}} \approx 23.800 \text{ mi/hr}$

c. During a second time at bat, the player hits another ball, which leaves the bat a distance of 43 inches from the player's shoulder. How fast (in miles per hour) would this ball be moving?

$\frac{10\pi \text{ RAD}}{3 \text{ sec}} \cdot \frac{2\pi(43 \text{ in})}{2\pi \text{ RAD}}$ (Circumference) $\cdot \frac{1 \text{ ft}}{12 \text{ in}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} = \frac{1{,}548{,}000\pi \text{ mi}}{190{,}080 \text{ hr}} \approx 25.585 \text{ mi/hr}$

Skillz Review Simplify the following.

1. $\frac{\frac{1}{2}}{\frac{5}{4}} \cdot \frac{4}{4} = \frac{2}{5}$

2. $\frac{\frac{1}{4}}{\frac{\sqrt{2}}{2}} \cdot \frac{4}{4} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{2 \cdot 2} = \frac{\sqrt{2}}{4}$

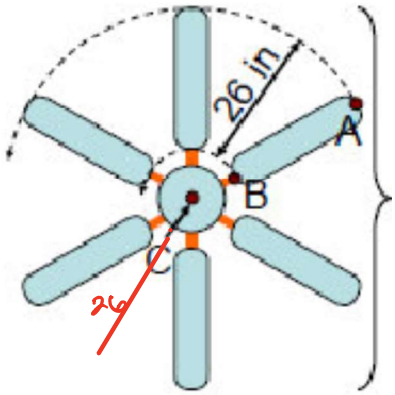
3. $\frac{5}{\left(\frac{\sqrt{3}}{2}\right)} \cdot \frac{2}{2} = \frac{10}{\sqrt{3}} = \frac{10\sqrt{3}}{3}$

4. $\frac{\left(\frac{\sqrt{3}}{2}\right)}{\sqrt{2}} \cdot \frac{2}{2} = \frac{2\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{6}}{2}$

25.58

APPLICATION

1. **The blades of a ceiling fan are 26 inches long, but the fan's entire diameter in 70 inches. It spins at a rate of 100 rpm.**
   a. What is the linear velocity of a point on the outer edge of the blade?



$70 - 52 = 18$

$\frac{100 \text{ Rev}}{1 \text{ min}} \cdot \frac{70\pi \text{ in}}{1 \text{ Rev}}$ (Circumference) $= 7000\pi \text{ in/min} \approx 21{,}991.149 \text{ in/min}$

   b. What is the linear velocity of a point on the inner edge of the blade?

$\frac{100 \text{ Rev}}{1 \text{ min}} \cdot \frac{18\pi \text{ in}}{1 \text{ Rev}}$ (Circumference) $= 1800\pi \text{ in/min} \approx 5654.867 \text{ in/min}$

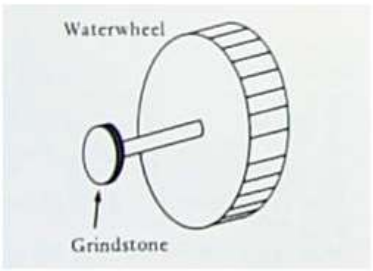
   c. What is the linear velocity of a point at the center of the fan?

$\frac{100 \text{ Rev}}{1 \text{ min}} \cdot \frac{0 \text{ in}}{1 \text{ Rev}}$ (Circumference) $= 0 \text{ in/min}$

   d. Do point A and point B have the same angular velocity OR the same linear velocity?

They have the same angular velocity.

2. **A waterwheel of diameter 12 feet turns at .3 radians per second.**
   a. What is the linear velocity of the rim?



$\frac{.3 \text{ RAD}}{1 \text{ sec}} \cdot \frac{12\pi \text{ feet}}{2\pi \text{ RAD}} = 1.8 \text{ ft/sec}$

   b. The wheel is connected by an axel to a grindstone of diameter 3 feet. What is the linear velocity of a point on the rim of the grindstone?

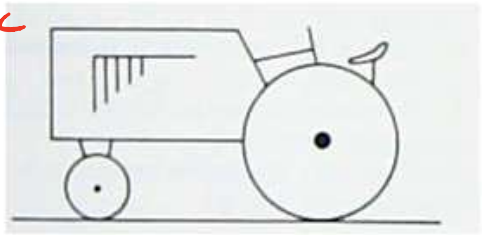
$\frac{.3 \text{ RAD}}{1 \text{ sec}} \cdot \frac{3\pi \text{ ft}}{2\pi \text{ RAD}} = \frac{.9}{2} \text{ ft/sec} = .45 \text{ ft/sec}$

   c. Do a point on the waterwheel and a point on the grindstone have the same angular velocity OR the same linear velocity?

They have the same angular velocity.

3. **The rear wheels of a tractor are 4 feet in diameter and turn at 20 rpm.**
   a. How fast is the tractor going in feet per second?

$\frac{20 \text{ Rev}}{1 \text{ min}} \cdot \frac{4\pi \text{ ft}}{1 \text{ Rev}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = \frac{80\pi}{60} = \frac{4\pi}{3} \text{ ft/sec} \approx 4.189 \text{ ft/sec}$



   b. The front wheels have a diameter of only 1.8 feet. What is the linear velocity of a point on their tire treads?

It must be the same as the rear wheels which is $\frac{4\pi}{3} \text{ ft/sec} \approx 4.189 \text{ ft/sec}$

   c. What is the angular velocity of the front wheels in rpm?

$\frac{4\pi \text{ ft}}{3 \text{ sec}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \cdot \frac{1 \text{ Rev}}{1.8\pi \text{ ft}} = \frac{80 \text{ Rev}}{1.8 \text{ min}} \approx 44.444 \text{ Rev/min}$

   d. Do a point on the rear wheel and a point on the front wheel have the same angular velocity OR the same linear velocity?

They have the same linear velocity.