Rational Exponents

Notes #
Omega 1

Scientific Notation:
A number is in scientific notation when it is in the form $a \bullet 10^n$, where $1 \leq a < 10$ and n is an integer.

Definitions of integral exponents:

If $n = 1$, $x^n = x$.

If $n > 1$, $x^n = \overbrace{x \bullet x \bullet x \ldots x}^{n \text{ factors}}$.

If $x \neq 0$, $x^0 = 1$.

If $x \neq 0$, $x^{-n} = \frac{1}{x^n}$

Properties of exponents: Suppose m and n are positive integers, and a and b are real numbers. Then the following properties hold.

Product property:	$a^m a^n = a^{m+n}$
Power of a power property:	$(a^m)^n = a^{mn}$
Power of a quotient property:	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, where $b \neq 0$
Power of a product property:	$(ab)^m = a^m b^m$
Quotient property:	$\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$

Definition of Rational Exponents: For any nonzero number b, and any integers m and n with $n > 1$,

$$b^{\frac{m}{n}} = \sqrt[n]{b^m} = \left(\sqrt[n]{b}\right)^m,$$

except when $\sqrt[n]{b}$ is not a real number.

Definition of $b^{\frac{1}{n}}$: For any real number $b \geq 0$ and any integer $n > 1$,

$$b^{\frac{1}{n}} = \sqrt[n]{b}.$$

This is also true when $b < 0$ and n is odd.

Ex A: Evaluate.

#1) $81^{\frac{1}{2}}$	#2) $27^{-\frac{2}{3}}$	#3) $7^{\frac{1}{4}} \bullet 7^{\frac{7}{4}}$
#4) $\sqrt[3]{125}$	#5) $(\sqrt[3]{343})^{-2}$	#6) $\frac{49^{\frac{5}{4}}}{49^{\frac{3}{4}}}$

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Ex B: Express using rational exponent.

#1) $\sqrt[4]{x}$	#2) $\sqrt{xy^3}$	#3) $\sqrt[3]{8x^3y^6}$
#4) $\sqrt{x^6y^3}$	#5) $\sqrt[3]{64x^9y^{15}}$	#6) $\sqrt[5]{15x^3y^{15}}$

Ex C: Express using radicals.

#1) $15^{\frac{1}{5}}$	#2) $x^{\frac{3}{4}}y^{\frac{1}{4}}$	#3) $15\,x^{\frac{1}{3}}y^{\frac{1}{5}}$

Ex D: Simplify.

#1) $(x^{-2})^4 \bullet x^8$	#2) $\sqrt{x^3y^2} \cdot \sqrt{x^4y^5}$	#3) $(5ac)^{\frac{1}{3}}(a^2c^3)^{\frac{1}{3}}$

Ex E: Word Problems.

#1) Red blood cells are circular-shaped cells that carry oxygen through your bloodstream. The diameter of a red blood cell is about $7.75 \bullet 10^{-7}$ m. Find the area of one of these cells. Express your answer in scientific notation.