## Logarithmic Functions

Notes \#
Omega 4
Exponential Function: A function of the form $y=a^{x}$, where $a$ is a positive real number.

Logarithm: The inverse of $y=a^{x}$ is $x=a^{y}$. In the function $x=a^{y}, y$ is called the logarithm. It is usually written $y=\log _{\mathrm{a}} \mathrm{x}$ and is read $y$ equals the log, base $a$, of $x$.

Logarithmic Function: A function in the form $y=\log _{a} x$.

Definition of Logarithmic Function:
The logarithmic function $y=\log _{a} x$, where $a>0$ and $a \neq 1$, is the inverse of the exponential function $y=a^{x}$.
Therefore, $y=\log _{a} x$ iff $x=a^{y}$.
Properties of Logarithms:
Suppose m and n are positive numbers, b is a positive number other than 1 , and p is any real number. Then the following properties are true.

Product Property: $\log _{\mathrm{b}} \mathrm{mn}=\log _{\mathrm{b}} \mathrm{m}+\log _{\mathrm{b}} \mathrm{n}$
Quotient Property: $\log _{\mathrm{b}} \frac{m}{n}=\log _{\mathrm{b}} \mathrm{m}-\log _{\mathrm{b}} \mathrm{n}$.

Power Property: $\log _{b} m^{p}=p \bullet \log _{b} m \quad \quad$ Property of Equality: If $\log _{b} m=\log _{b} n$, then $m=n$.
$\underline{\text { Log Identity 1: }} \log _{a} a^{x}=x$
$\underline{\text { Log Identity } 2:} a^{\log _{a} x}=x$

Ex A: Write each equation in logarithmic form.

| $\# 1) \quad 4^{3}=64$ | $\# 2) \quad 6^{-2}=\frac{1}{36}$ | \#3) $49^{\frac{1}{2}}=7$ |  |
| :--- | :--- | :--- | :--- |
| $\# 1$ | $\# 2$ |  | \#3 |

Ex B: Write each equation in exponential form.

| $\# 1) \quad \log _{27} 3=\frac{1}{3}$ | $\# 2) \quad \log _{16} 4=1 / 2$ | \#3) $\log _{9} 27=\frac{3}{2}$ |
| :--- | :--- | :--- | :--- | :--- |
| $\# 1$ | $\# 2$ | $\# 3$ |

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Ex C: Evaluate each expression.

| \#1) | $\log _{2} 32$ |  | \#2) | $\log _{10} 1000$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | \#1 |  |  | \#2 |
| \#3) | $\log _{7} \frac{1}{343}$ |  | \#4) | $\log _{10} 0.0001$ |  |
|  |  | \#3 |  |  | \#4 |
| \#5) | $\log _{2} \frac{1}{32}$ |  | \#6) | $5^{3 \log _{5} 2}$ |  |
|  |  | \#5 |  |  | \#6 |

Ex D: Solve each equation.

| \#1) | $\log _{6} \mathrm{x}+\log _{6} 9=\log _{6} 54$ | \#2) $\quad \log _{7} \mathrm{n}=\frac{2}{3} \log _{7} 8$ |  |
| :---: | :---: | :---: | :---: |

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Ex D: Solve each equation.

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| \#3) $4 \log _{2} \mathrm{x}+\log _{2} 5=\log _{2} 405$ | \#4) $\log _{8} 48-\log _{8} \mathrm{w}=\log _{8} 4$ |
| :---: | :---: | :---: | :---: |

Ex E: Word problems.
\#1) If the population of 100 bacteria doubles every fifteen minutes, how long will it take for the population to reach 12,800 ?

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Ex F: Graph each equation or inequality.

$$
\# 1) \quad y=\log _{3}(x+1)
$$

\#1) Change to exponential form.
\#2) Solve your equation for x instead of $y$.
\#3) When doing your table substitute numbers into y instead of $x$.

Recall that exponential graphs have a horizontal asymptote. Therefore logarithmic graphs (inverse of exponential) have a vertical asymptote.
\#2) $\quad \mathrm{y}<\log _{2} \mathrm{x}$

