Exponential & Logarithmic Functions
6 – Exponential & Logarithmic Equations

Half-Life: The time it takes for one-half of a quantity of a radioactive element to decay.

Formula for Growth and Decay in Nature:

$y = y_0 c^{\frac{t}{T}}$, where y is the final amount, y_0 is the initial amount, c is the constant of proportionality, t is time, and T is the time per cycle of c. When using half-life, $c = ½$ and T is the half-life.

(If $0 < c < 1$, the population decays over time. If $c > 1$, the population grows.)

(Ex. If something triples every 7 days, $c = 3$ and $T = 7$)

Exponential Equations:
Equations in which variables appear as exponents.

Change of Base Formula:
Suppose a, b, and n are positive numbers, and neither a nor b is 1. Then the following equation is true.

$$\log_a n = \frac{\log_b n}{\log_b a}$$

Ex A: Solve each equation or inequality by using logarithms. Round solutions to the nearest hundredth.

#1) $\log_8 ½ = x$

#2) $\log_{27} ½ = x$

#3) $10^{x-3} \geq 52$

#4) $5^x \leq 7\sqrt{6}$

#5) $2^x = 14$

#6) $\log_5 7 = x$

#7) $2^x > \sqrt{3^{x-2}}$

#8) $6^{x^2-2} < 48$

#9) $\log_x 9 > 3$

Ex B: Use a graphing calculator to solve each equation or inequality by graphing. Round solutions to the nearest hundredth.

#1) $5^x = 12^{3-x}$

#2) $4^{2x-5} \leq -3^{x-3}$

#3) $\log(3x-1) \leq 2^{x-7}$

Ex C: Word Problems.

#1) Stonehenge is an ancient megalithic site in southern England. Some believe it was designed to make astronomical observations, but archaeologists cannot agree on the reason that Stonehenge was constructed. Charcoal samples taken from a series of holes at Stonehenge have about 0.63 pounds of carbon-14 in a one-pound sample. Estimate the age of the charcoal pits at Stonehenge. Assume that the half-life of carbon-14 is 5730 years.

#2) Under ideal conditions, the population of a single-celled organism in a pond will double in 5 days. How much time will it take for the original population to increase seven times?