Trig Identities & Equations
Unit 11.1 – Basic Trig IDs

Reciprocal Identities: The following trig identities hold for all values of A except those for which any function is undefined.

$\sin A = \frac{1}{\csc A}$ $\quad\quad$ $\csc A = \frac{1}{\sin A}$

$\cos A = \frac{1}{\sec A}$ $\quad\quad$ $\sec A = \frac{1}{\cos A}$

$\tan A = \frac{1}{\cot A}$ $\quad\quad$ $\cot A = \frac{1}{\tan A}$

Quotient Identities: The following trig identities hold for all values of A except those for which any function is undefined.

$\tan A = \frac{\sin A}{\cos A}$ $\quad\quad$ $\cot A = \frac{\cos A}{\sin A}$

Pythagorean Identities: The following trig identities hold for all values of A except those for which any function is undefined.

$\sin^2 A + \cos^2 A = 1$

$\tan^2 A + 1 = \sec^2 A$

$1 + \cot^2 A = \csc^2 A$

Properties of Equality for Real Numbers

Multiplication (& Division) Property
If x = y, then xz = yz

Addition (& Subtraction) Property
If x = y, then x + z = y + z

Distributive Property
x(y + z) = xy + xz
(y + z)x = xy + xz

Reflexive Property
x = x

Symmetric Property
If x = y, then y = x

Transitive Property
If x = y and y = z, then x = z.

Substitution Property
If x = y, then x may replace y in any equation.

Root Property
If x = y + z and n is an odd integer, then
$\sqrt[n]{x} = \sqrt[n]{y+z}$.
If x = y + z and n is an even integer, then
$\sqrt[n]{x} = \pm\sqrt[n]{y+z}$.

Exponential Property
If x = y + z and n is an integer, then $x^n = (y+z)^n$.

Addition Properties of Equality

Commutative Property
x + y = y + x

Associative Property
(x + y) + z = x + (y + z)

Additive Identity
x + 0 = x

Additive Inverse Property
x + (-x) = 0

Multiplication Properties of Equality

Commutative Property
xy = yx

Associative Property
(xy)z = x(yz)

Multiplicative Identity
x(1) = x

Multiplicative Inverse Property
$x \cdot \frac{1}{x} = 1$

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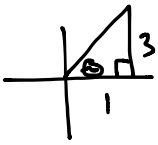
Ex A: Solve for values of θ between 0º and 90º.

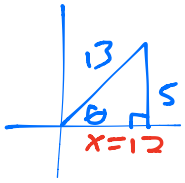
#1) If tan θ =3, find cot θ.

$\cot\theta = \frac{1}{3}$



Draw a reference angle in Quad I. Make a right triangle. If needed find the length of the third side of the triangle using Pythagorean theorem. Write your new ratio.

#2) If $\sin\theta = \frac{5}{13}$, find cos θ.



$\cos\theta = \frac{12}{13}$

$x^2+y^2=r^2$

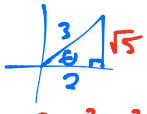
$x^2+(5)^2=(13)^2$

$x^2+25=169$

$x^2=144$

$x=\pm 12$

#3) If $\cos\theta = \frac{2}{3}$, find tan θ.



$\tan\theta = \frac{\sqrt{5}}{2}$

$x^2+y^2=r^2$

$(2)^2+y^2=(3)^2$

$4+y^2=9$

$y^2=5$

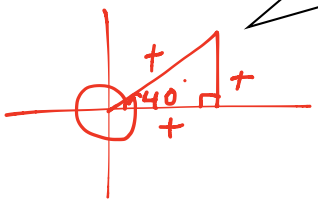
$y=\pm\sqrt{5}$

Ex B: Express each value as a function of an angle in Quadrant I.

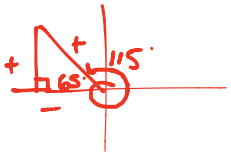
(You may want to add or subtract multiples of 360º to find a more appropriate coterminal angle.) First find the reference angle and draw a right triangle. Label the x, y, and r values with a + or -. If your ratio has two values of the same sign, then the function is positive. If your ratio has two values with different signs, then the function is negative.

#1) sin 400º

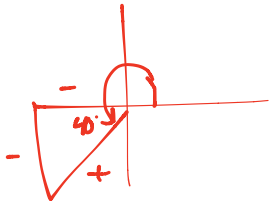
$\sin 40^\circ$



#2) tan 475º $= -\tan 65^\circ$



#3) cos 220º $= -\cos 40^\circ$



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Ex C: Write an expression for each of the following in terms of the given function.

#1) sec θ in terms of cot θ

$\sec^2\theta = \tan^2\theta + 1$

$\sec\theta = \pm\sqrt{\tan^2\theta + 1}$

$= \pm\sqrt{\frac{1}{\cot^2\theta} + 1}$

$= \pm\sqrt{\frac{1}{\cot^2\theta} + \frac{\cot^2\theta}{\cot^2\theta}}$

$= \pm\sqrt{\frac{1+\cot^2\theta}{\cot^2\theta}}$

$= \frac{\pm\sqrt{1+\cot^2\theta}}{\sqrt{\cot^2\theta}}$

$\sec\theta = \frac{\pm\sqrt{1+\cot^2\theta}}{\cot\theta}$

#2) csc θ in terms of cos θ

$\sin^2\theta + \cos^2\theta = 1$

$\frac{1}{\csc^2\theta} + \cos^2\theta = 1$

$\frac{1}{\csc^2\theta} = 1 - \cos^2\theta$

$1 = (1-\cos^2\theta)\csc^2\theta$

$\frac{1}{1-\cos^2\theta} = \csc^2\theta$

$\pm\sqrt{\frac{1}{1-\cos^2\theta}} = \csc\theta$

$\pm\frac{1}{\sqrt{1-\cos^2\theta}} \cdot \frac{\sqrt{1-\cos^2\theta}}{\sqrt{1-\cos^2\theta}} = \csc\theta$

$\pm\frac{\sqrt{1-\cos^2\theta}}{1-\cos^2\theta} = \csc\theta$

Ex D: Simplify.

#1) $\csc^2\theta - \cot^2\theta$

$= (1+\cot^2\theta) - \cot^2\theta$

$= 1 + \cot^2\theta - \cot^2\theta$

$= 1$

#2) $\frac{\sin^2 x + \cos^2 x}{\cos^2 x}$

$= \frac{1}{\cos^2 x}$

$= \sec^2 x$

#3) cos y csc y

$= \cos y \cdot \frac{1}{\sin y}$

$= \frac{\cos y}{\sin y}$

$= \cot y$

#4) $\frac{\tan z}{\sin z}$

$= \frac{\frac{\sin z}{\cos z}}{\sin z} \cdot \frac{\cos z}{\cos z}$

$= \frac{\sin z}{\sin z \cos z}$

$= \frac{1}{\cos z}$

$= \sec z$

#5) tan A csc A

$= \frac{\sin A}{\cos A} \cdot \frac{1}{\sin A}$

$= \frac{1}{\cos A}$

$= \sec A$

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