Trig Identities & Equations
Unit 11.3 – Sum & Difference IDs

Sum and Difference Identities for the Cosine Function: If α and β represent the measure of two angles, then the following identities hold for all values of α and β.

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

Sum and Difference Identities for the Sine Function: If α and β represent the measure of two angles, then the following identities hold for all values of α and β.

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

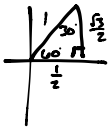
Sum and Difference Identities for the Tangent Function: If α and β represent the measure of two angles, then the following identities hold for all values of α and β.

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

Ex A: Use the sum and difference identities to find the exact value of each function.

#1) Find cos 105º from values of functions of 60º and 45º.



$\cos 105° = \cos(60° + 45°)$

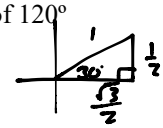
$= \cos 60°\cos 45° - \sin 60°\sin 45°$

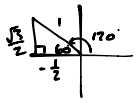
$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$

$\cos 105° = \frac{\sqrt{2} - \sqrt{6}}{4}$

#2) Find sin 150º from values of functions of 120º and 30º.





$\sin 150° = \sin(120° + 30°)$

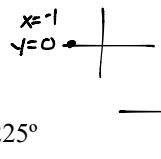
$= \sin 120°\cos 30° + \cos 120°\sin 30°$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{3}{4} - \frac{1}{4}$

$= \frac{2}{4}$

$\sin 150° = \frac{1}{2}$



#3) cos 225°

$\cos 225° = \cos(180° + 45°)$

$= \cos 180° \cos 45° - \sin 180° \sin 45°$

$= (-1)\left(\frac{\sqrt{2}}{2}\right) - (0)\left(\frac{\sqrt{2}}{2}\right)$

$\cos 225° = -\frac{\sqrt{2}}{2}$

Ex B: If α and β are the measures of two first quadrant angles, find the exact value of each function.

#1) If $\sin\alpha = \frac{5}{13}$ and $\cos\beta = \frac{4}{5}$, find $\cos(\alpha + \beta)$.

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \left(\frac{12}{13}\right)\left(\frac{4}{5}\right) - \left(\frac{5}{13}\right)\left(\frac{3}{5}\right)$

$= \frac{48}{65} - \frac{15}{65}$

$\cos(\alpha+\beta) = \frac{33}{65}$

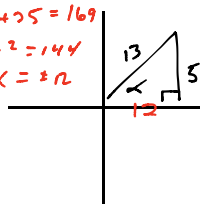
$x^2+y^2=r^2$

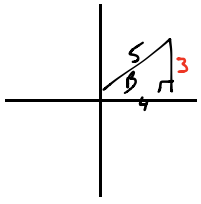
$x^2+(5)^2=(13)^2$

$x^2+25=169$

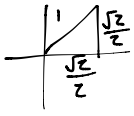
$x^2=144$

$x=\pm 12$

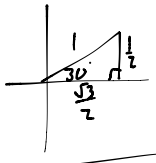




Pythag Triple



#4) $\tan 15° = \tan(45° - 30°)$



$= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°}$

$= \frac{1 - \frac{\sqrt{3}}{3}}{1 + (1)\left(\frac{\sqrt{3}}{3}\right)}$

$= \frac{\frac{3}{3} - \frac{\sqrt{3}}{3}}{\frac{3}{3} + \frac{\sqrt{3}}{3}}$

$= \frac{\frac{3-\sqrt{3}}{3}}{\frac{3+\sqrt{3}}{3}}$

$= \frac{3-\sqrt{3}}{3} \cdot \frac{3}{3+\sqrt{3}}$

$= \frac{3-\sqrt{3}}{3+\sqrt{3}} \cdot \frac{(3-\sqrt{3})}{(3-\sqrt{3})}$

$= \frac{9-6\sqrt{3}-3}{9-3}$

$= \frac{12-6\sqrt{3}}{6}$

$= \frac{6(2-\sqrt{3})}{6}$

$= 2-\sqrt{3}$

$\tan 30° = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2}\cdot\frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\tan 30° = \frac{\sqrt{3}}{3}$

#2) If $\cos\alpha = \frac{5}{13}$ and $\cos\beta = \frac{35}{37}$, find $\tan(\alpha + \beta)$.

$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$= \frac{\left(\frac{12}{5}\right) + \left(\frac{12}{35}\right)}{1 - \left(\frac{12}{5}\right)\left(\frac{12}{35}\right)}$

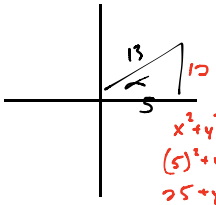
$= \frac{\frac{12}{5} + \frac{12}{35}}{1 - \left(\frac{12}{5}\right)\left(\frac{12}{35}\right)}$

$= \frac{\frac{84}{35} + \frac{12}{35}}{\frac{175}{175} - \frac{144}{175}}$

$= \frac{\frac{96}{35}}{\frac{31}{175}}$

$= \frac{96}{35} \cdot \frac{175}{31}$

$= \frac{480}{31}$



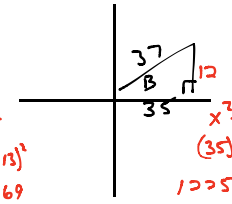
$x^2+y^2=r^2$

$(5)^2+y^2=(13)^2$

$25+y^2=169$

$y^2=144$

$y=\pm 12$



$x^2+y^2=r^2$

$(35)^2+y^2=(37)^2$

$1225+y^2=1369$

$y^2=144$

$y=\pm 12$

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#3) If $\csc\alpha = \frac{13}{5}$ and $\tan\beta = \frac{3}{4}$, find $\tan(\alpha - \beta)$.

$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

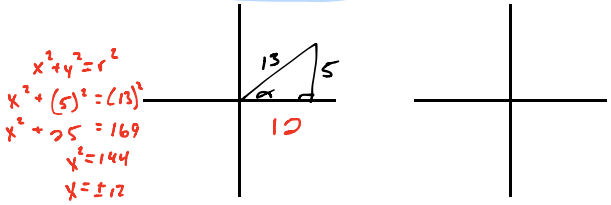
$$= \frac{\left(\frac{5}{12}\right) - \left(\frac{3}{4}\right)}{1 + \left(\frac{5}{12}\right)\left(\frac{3}{4}\right)}$$

$$= \frac{\frac{5}{12} - \frac{9}{12}}{\frac{16}{16} + \frac{5}{16}}$$

$$= \frac{\frac{-4}{12}}{\frac{21}{16}}$$

$$= -\frac{1}{3}\cdot\frac{16}{21}$$

$$\tan(\alpha-\beta) = \frac{-16}{63}$$



$x^2+y^2=r^2$
$x^2+(5)^2=(13)^2$
$x^2+25=169$
$x^2=144$
$x=\pm 12$

Ex C: Verify that each of the following is an identity.

#1) $\sin(180° - \theta) = \sin\theta$

$\sin 180°\cos\theta - \cos 180°\sin\theta =$

$0\cdot\cos\theta - (-1)\sin\theta =$

$0 + \sin\theta =$

$\sin\theta = \sin\theta$

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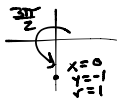
#2) $\cos\left(\frac{3\pi}{2}+\theta\right)=\sin\theta$



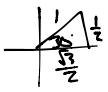
$\cos\frac{3\pi}{2}\cdot\cos\theta-\sin\frac{3\pi}{2}\sin\theta=$

$0\cdot\cos\theta-(-1)\sin\theta=$

$0+\sin\theta=$

$\sin\theta=\sin\theta$

#3) $\cos(30°-x)+\cos(30°+x)=\sqrt{3}\cos x$



$\cos30°\cos x+\sin30\sin x+\cos30°\cos x-\sin30°\sin x=$

$2\cos30°\cos x=$

$2\left(\frac{\sqrt{3}}{2}\right)\cos x=$

$\sqrt{3}\cos x=\sqrt{3}\cos x$