Trig Identities & Equations
Unit 11.4 – Double & Half-Angle IDs

Double-Angle Identities:
If θ represents the measure of an angle, then the following identities hold for all values of θ.

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$
$\cos 2\theta = 2\cos^2\theta - 1$
$\cos 2\theta = 1 - 2\sin^2\theta$

$$\tan 2\theta = \frac{2\tan\theta}{1 - tan^2\theta}$$

Half-Angle Identities:
If α represents the measure of an angle, then the following identities hold for all values of α.

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$\tan\frac{\alpha}{2} = \pm\frac{\sin\alpha}{1+\cos\alpha} \quad , \cos\alpha \neq -1$$

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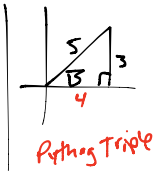
Ex A: If $\sin B = \frac{3}{5}$ and B is in the first quadrant, find each value.

#1) sin 2B

$= 2\sin B \cos B$

$= 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)$

$= \frac{24}{25}$



Pythag Triple

#2) cos 2B

$= \cos^2 B - \sin^2 B$

$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$= \frac{16}{25} - \frac{9}{25}$

$= \frac{7}{25}$

Ex A: If $\sin B = \frac{3}{5}$ and B is in the first quadrant, find each value.

#3) $\tan\frac{B}{2}$

$= \pm\frac{\sin B}{1+\cos B}$

$= +\frac{\frac{3}{5}}{\frac{5}{5}+\frac{4}{5}}$

$= \frac{\frac{3}{5}}{\frac{9}{5}} \cdot \frac{5}{5}$

$= \frac{3}{9}$

$= \frac{1}{3}$

If $0° < B < 90°$ then $0° < \frac{B}{2} < 45°$ thus $\frac{B}{2}$ is Quad I

Ex B: Use a half-angle identity to find each value.

#1) sin 22°30'

$= \sin\left(\frac{45°}{2}\right)$

$= \pm\sqrt{\frac{1-\cos 45°}{2}}$

$= +\sqrt{\frac{\frac{2}{2}-\frac{\sqrt{2}}{2}}{2}}$

$= \sqrt{\frac{\frac{2-\sqrt{2}}{2}}{2}\cdot\frac{2}{2}}$

$= \sqrt{\frac{2-\sqrt{2}}{4}}$

$= \frac{\sqrt{2-\sqrt{2}}}{\sqrt{4}}$

$= \frac{\sqrt{2-\sqrt{2}}}{2}$

Since 22°30' is in Quad I, $\sin 22°30' = +$

Ex B: Use a half-angle identity to find each value.

#2) $\cos\frac{19\pi}{12}$

$= \cos\frac{\frac{19\pi}{6}}{2}$

$= \pm\sqrt{\frac{1+\cos\frac{19\pi}{6}}{2}}$

$= +\sqrt{\frac{\frac{2}{2}+\left(\frac{-\sqrt{3}}{2}\right)}{2}}$

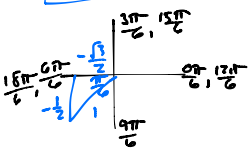
$= \sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}\cdot\frac{2}{2}}$

$= \sqrt{\frac{2-\sqrt{3}}{4}}$

$= \frac{\sqrt{2-\sqrt{3}}}{\sqrt{4}}$

$= \frac{\sqrt{2-\sqrt{3}}}{2}$

$\frac{19\pi}{12}$ is in Quad IV so $\cos\frac{19\pi}{12}$ is +



#3) $\tan 22°30'$

$= \tan\left(\frac{45°}{2}\right)$

$= \pm\frac{\sin 45°}{1+\cos 45°}$

$= +\frac{\frac{\sqrt{2}}{2}}{\frac{2}{2}+\frac{\sqrt{2}}{2}}$

$= \frac{\frac{\sqrt{2}}{2}}{\frac{2+\sqrt{2}}{2}}\cdot\frac{2}{2}$

$= \frac{\sqrt{2}}{2+\sqrt{2}}\cdot\frac{(2-\sqrt{2})}{(2-\sqrt{2})}$

$= \frac{2\sqrt{2}-2}{4-2}$

$= \frac{2(\sqrt{2}-1)}{2}$

$= \sqrt{2}-1$

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Ex C: Verify that each of the following is an identity.

#1) $\frac{1}{2}\sin 2A = \frac{\tan A}{1+\tan^2 A}$

$\frac{1}{2}(2\sin A\cos A) =$

$\sin A\cos A = \dfrac{\frac{\sin A}{\cos A}}{\frac{\cos^2 A}{\cos^2 A}+\frac{\sin^2 A}{\cos^2 A}}$

$= \dfrac{\frac{\sin A}{\cos A}}{\frac{\cos^2 A+\sin^2 A}{\cos^2 A}}$

$= \dfrac{\frac{\sin A}{\cos A}}{\frac{1}{\cos^2 A}} \cdot \dfrac{\cos^2 A}{\cos^2 A}$

$\sin A\cos A = \sin A\cos A$

#2) $\tan 2x\tan x + 2 = \frac{\tan 2x}{\tan x}$

$\frac{2\tan x}{1-\tan^2 x}\tan x + 2 =$

$\frac{2\tan^2 x}{1-\tan^2 x} + 2 =$

$\frac{2\tan^2 x}{1-\tan^2 x} + \frac{2(1-\tan^2 x)}{1-\tan^2 x} =$

$\frac{2\tan^2 x + 2 - 2\tan^2 x}{1-\tan^2 x} =$

$\frac{2}{1-\tan^2 x} =$

$\frac{2}{1-\tan^2 x}\cdot\frac{\tan x}{\tan x} =$

$\frac{2\tan x}{1-\tan^2 x}\cdot\frac{1}{\tan x} =$

$\tan 2x \cdot \frac{1}{\tan x} =$

$\frac{\tan 2x}{\tan x} = \frac{\tan 2x}{\tan x}$