## Solving Triangles

## 12.1 - Law of Sines

Law of Sines: Let $\triangle A B C$ be any triangle with $a, b$, and $c$ representing the measures of the sides opposite the angles with measures $A, B$, and $C$ respectively. Then, the following is true.

$$
\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}
$$

When Do I use Law of Sines?
If you are given one of the following cases:
Case \#1
AAS (two angles and a side opposite one of those angles.)


Case \#2
ASS (two sides and an angle opposite one of those sides.)


Ambiguous Case for Law of Sines: When you are given ASS:

Use Law of Sines to find a missing angle. If the first angle you are trying to find doesn't exist, there is 0 solutions. If the angle is $90^{\circ}$, there is 1 solution. If the angle is acute, there might be 2 solutions.

Ex A: Determine the number of possible solutions. If a solution exists, solve the triangle. Round angle measures to the nearest minute and side measures to the nearest tenth.
\#1) $\mathrm{b}=20, \mathrm{C}=30^{\circ}, \mathrm{c}=10$


$$
\begin{aligned}
& \text { (2) } \begin{array}{l}
\frac{\sin 30^{\circ}}{10}=\frac{\sin B}{20} \\
\frac{20 \sin 30^{\circ}}{10}=\sin B \\
\sin ^{-1}\left(\frac{20 \sin 30^{\circ}}{10}\right)=B \\
90^{\circ}=B \\
\text { Since Ass ana } B=90^{\circ}, \\
\text { here is one solution }
\end{array}
\end{aligned}
$$

$$
\text { (3) } \begin{gathered}
m \angle A+90^{\circ}+30^{\circ}=180^{\circ} \\
m \angle A+130^{\circ}=180^{\circ} \\
m \angle A=60^{\circ}
\end{gathered}
$$



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\#3) $\mathrm{b}=20, \mathrm{C}=30^{\circ}, \mathrm{c}=7$

(2)

$$
\begin{gathered}
\frac{\sin 30^{\circ}}{7}=\frac{\sin B}{20} \\
\frac{20 \sin 30^{\circ}}{7}=\sin B \\
\sin ^{-1}\left(\frac{20 \sin 30^{\circ}}{7}\right)=B \\
{[\text { rror }=B}
\end{gathered}
$$

Since $A S S$ and $m \angle B$ is error, there is no solution
\#4) $\mathrm{b}=20, \mathrm{C}=30^{\circ}, \mathrm{c}=13$

(3)

$$
\begin{aligned}
m \angle A+30^{\circ}+50^{\circ} 17^{\prime} & =180^{\circ} \\
m \angle A+80^{\circ} 17^{\prime} & =180^{\circ} \\
m \angle A & =99^{\circ} 43^{\prime}
\end{aligned}
$$

$$
\left(\begin{array}{c}
\frac{\sin 30^{\circ}}{13}=\frac{\sin B}{20} \\
\frac{20 \sin 30^{\circ}}{13}=\sin B \\
\sin ^{\circ}\left(\frac{20 \sin 30^{\circ}}{13}\right)=B \\
50^{\circ} 17^{\circ}=B
\end{array}\right.
$$

(4)

$$
\begin{aligned}
& \frac{\sin 30^{\circ}}{13}=\frac{\sin 99^{\circ} 43^{\prime}}{a} \\
& a \cdot \sin 30^{\circ}=13 \sin 99^{\circ} 43^{\circ} \\
& a=\frac{13 \sin 99^{\circ} 43^{\circ}}{\sin 30^{\circ}} \\
& a=25.6
\end{aligned}
$$

$$
\begin{aligned}
& 2 \text { solutions becanke Ass. } \\
& \text { and } m \angle B \text { is accute }
\end{aligned}
$$

Soluhn 1

Solctie 2
(ㄱ)

$$
\text { 66) } \begin{aligned}
& m \angle B^{\prime}+m \angle B=180^{\circ} \\
& m \angle B^{\prime}+50^{\circ} 17^{\prime}=180^{\circ} \\
& m \angle B^{\prime}=129^{\circ} 43^{\circ}
\end{aligned}
$$

$$
\begin{gathered}
m \angle A^{\prime}+129^{\circ} 43^{\prime}+30^{\circ}=180 \\
m \angle A^{\prime}+159^{\circ} 43^{\circ}=180^{\circ} \\
m \angle A^{\prime}=20^{\circ} 17^{\circ} \\
\frac{\sin 30^{\circ}}{13}=\frac{\sin 20^{\circ} 17^{\prime}}{a^{\prime}} \\
a^{\prime} \cdot \sin 30^{\circ}=13 \sin 20^{\circ} n^{\prime} \\
a^{\prime}=\frac{13 \sin 20^{\circ} 17^{\prime}}{\sin 30^{\circ}} \\
a^{\prime} \approx 9.0
\end{gathered}
$$

(8)

| $\frac{2}{3}+\frac{1}{4}=$ | $\frac{2 x}{3}+\frac{x}{4}=$ | $\frac{2 \sin x}{3}+\frac{\sin x}{4}=$ |
| :--- | :--- | :--- |
| $\left(\frac{2}{3}\right)^{2}=$ | $\left(\frac{2 x}{3}\right)^{2}=$ | $\left(\frac{2 \sin x}{3}\right)^{2}=$ |

