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Solving Triangles
12.2 – Law of Cosines

Law of Cosines: Let ΔXYZ be any triangle with *x, y,* and *z* representing the measures of the sides opposite the angles with measures *X, Y,* and *Z* respectively. Then, the following is true.

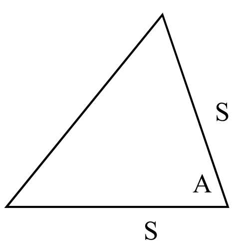
$$x^2 = y^2 + z^2 - 2yz\cos X$$
$$y^2 = x^2 + z^2 - 2xz\cos Y$$
$$z^2 = x^2 + y^2 - 2xy\cos Z$$

Once you find the 4th measure of the triangle, you can use Law of Cosines or Law of Sines to find the 5th measure.

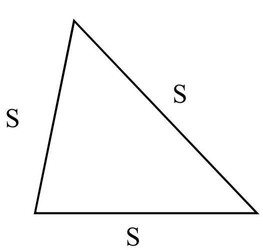
<u>When Do I use Law of Cosines?</u>

If you are given one of the following cases:

Case #1
SAS (two sides and included angle)



Case #2
SSS (three sides)

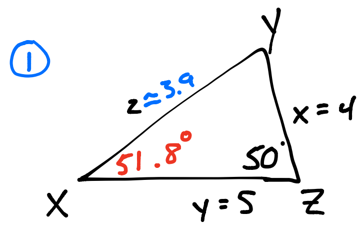


If you are given SSS, you must find the largest angle with cosine.

Ex A: Solve each triangle. Round angle measures ~~to the nearest minute~~ and side measures to the nearest tenth.

#1) $x = 4$, $y = 5$, $m\angle Z = 50^\circ$

1



2

$z^2 = x^2 + y^2 - 2xy\cos Z$

$z^2 = (4)^2 + (5)^2 - 2(4)(5)\cos 50^\circ$

$z^2 = 16 + 25 - 40\cos 50^\circ$

$z^2 = 41 - 40\cos 50^\circ$

$z = \pm\sqrt{41 - 40\cos 50^\circ}$

$z \approx 3.9$

3

$\dfrac{\sin 50^\circ}{3.9} \approx \dfrac{\sin X}{4}$

$\dfrac{4\sin 50^\circ}{3.9} \approx \sin X$

$\sin^{-1}\left(\dfrac{4\sin 50^\circ}{3.9}\right) \approx X$

$51.8^\circ \approx X$

4

$m\angle Y + 51.8^\circ + 50^\circ = 180^\circ$

$m\angle Y + 101.8^\circ = 180^\circ$

$m\angle Y = 78.2^\circ$

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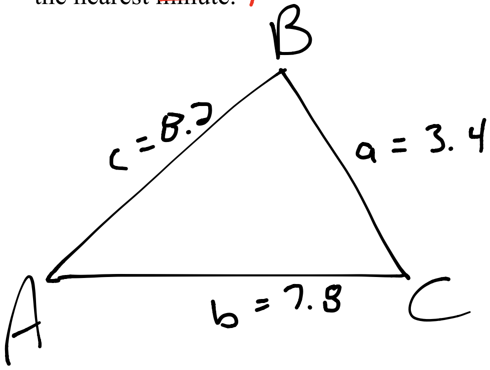
#2) The sides of a triangle measure 7.8 cm, 8.2 cm, and 3.4 cm. Find the measure of the smallest angle to the nearest ~~minute.~~ tenth



$a^2 = b^2 + c^2 - 2bc\cos A$

$(3.4)^2 = (7.8)^2 + (8.2)^2 - 2(7.8)(8.2)\cos A$

$11.56 = 60.84 + 67.24 - 127.92\cos A$

$11.56 = 128.08 - 127.92\cos A$

$-116.52 = -127.92\cos A$

$\frac{116.52}{127.92} = \cos A$

$\cos^{-1}\left(\frac{116.52}{127.92}\right) = A$

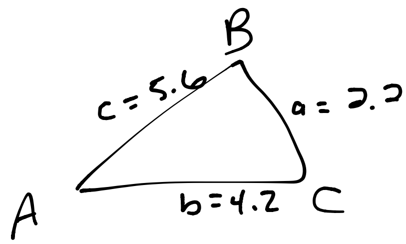
$24.4^\circ \approx A$

The smallest angle is about 24.4°

Draw the triangle.

Remember the smallest angle is opposite the smallest side.

#3) The sides of a triangle measure 4.2 cm, 5.6 cm, and 2.2 cm. Find the measure of the largest angle to the nearest ~~minute~~. tenth



$c^2 = a^2 + b^2 - 2ab\cos C$

$(5.6)^2 = (2.2)^2 + (4.2)^2 - 2(2.2)(4.2)\cos C$

$31.36 = 4.84 + 17.64 - 18.48\cos C$

$31.36 = 22.48 - 18.48\cos C$

$8.88 = -18.48\cos C$

$\frac{8.88}{-18.48} = \cos C$

$118.7^\circ \approx C$

The largest angle is about 118.7°