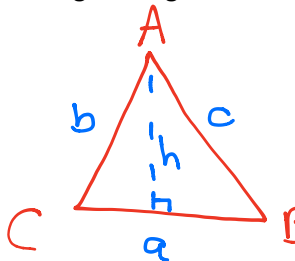


## 12.3 Area of Triangles

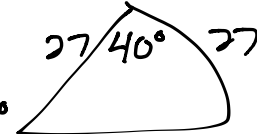
1

The side lengths of a triangular traffic sign are approximately 27 cm, 27 cm and ????. The angle between the given legs is  $40^\circ$ . Approximate the area of the sign. Round your answer to the nearest whole  $\text{cm}^2$ .



$\sin C = \frac{h}{b}$   
 $b \sin C = h$   
 $A_{\Delta} = \frac{1}{2} b h$   
 $A_{\Delta} = \frac{1}{2} a \cdot b \sin C$

$A_{\Delta} = \frac{1}{2} bc \sin A$   
 $= \frac{1}{2} (27)(27) \sin 40^\circ$   
 $= 364.5 \sin 40^\circ$   
 $A_{\Delta} \approx 234.296 \text{ cm}^2$



### Area of Triangles

Let  $\Delta ABC$  be any triangle with  $a$ ,  $b$ , and  $c$  representing the measures of the sides opposite the angles with measurements  $A$ ,  $B$  and  $C$  respectively. Then the area of the triangle equals:

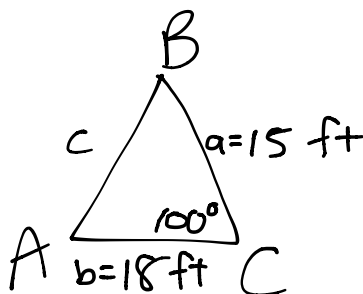
#### Hero's (Heron's) Formula

$$A = \frac{1}{2} ab \sin C$$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where  $s$  is the *semiperimeter* of the triangle.

**Example 1.** Find the area of  $\Delta ABC$  if  $a = 15$  ft.,  $b = 18$  ft. and  $m\angle C = 100^\circ$ .



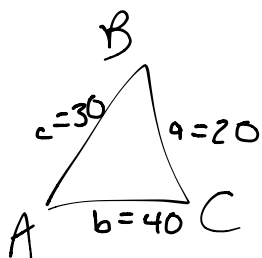
$$A_{\Delta} = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2} (15)(18) \sin 100^\circ$$

$$= 135 \sin 100^\circ$$

$$A_{\Delta} \approx 132.949 \text{ ft}^2$$

**Example 2.** Find the area of  $\Delta ABC$  if  $a = 20$ ,  $b = 40$  and  $c = 30$ .



$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{45(45-20)(45-40)(45-30)}$$

$$= \sqrt{45(25)(5)(15)}$$

$$= \sqrt{84,375}$$

$$A = 290.474 \text{ un}$$

$$s = \frac{1}{2} (a+b+c)$$

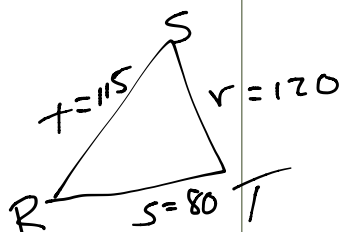
$$= \frac{1}{2} (20+40+30)$$

$$= \frac{1}{2} (90)$$

$$s = 45$$

# 12.3 Area of Triangles

You try! Find the following areas:



3. Find the area of  $\Delta RST$  if  $r = 120$ ,  $s = 80$  and  $t = 115$ .

$$S = \frac{1}{2}(a+b+c)$$

$$= \frac{1}{2}(80+115+120)$$

$$= \frac{1}{2}(315)$$

$$S = 157.5$$

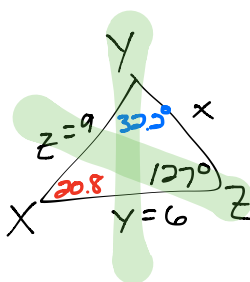
$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{157.5(157.5-120)(157.5-80)(157.5-115)}$$

$$= \sqrt{157.5(37.5)(77.5)(42.5)}$$

$$= \sqrt{19453710.9375}$$

$$A \approx 4,410.636 \text{ un}^2$$



4. Find the area of  $\Delta XYZ$  if  $y = 6 \text{ m}$ ,  $m\angle Z = 127^\circ$  and  $z = 9 \text{ m}$ .

$$\frac{\sin 127^\circ}{9} = \frac{\sin Y}{6}$$

$$\frac{6 \sin 127^\circ}{9} = \sin Y$$

$$\sin^{-1}\left(\frac{6 \sin 127^\circ}{9}\right) = Y$$

$$32.2^\circ = Y$$

$$127^\circ + 32.2^\circ + X = 180^\circ$$

$$159.2^\circ + X = 180^\circ$$

$$X = 20.8^\circ$$

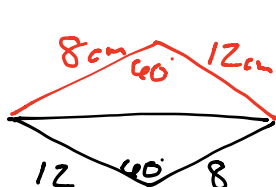
$$A = \frac{1}{2}yz \sin X$$

$$= \frac{1}{2}(6)(9) \sin 20.8^\circ$$

$$= 27 \sin 20.8^\circ$$

$$A \approx 9.6 \text{ m}^2$$

5. The adjacent sides of a parallelogram measure 8 cm and 12 cm and one angle between them measures  $60^\circ$ . Find the area of the parallelogram.



$$A_{\square} = 2A_{\Delta}$$

$$= 2\left(\frac{1}{2}bc \sin A\right)$$

$$= (8)(12) \sin 60^\circ$$

$$= 96 \cdot \frac{\sqrt{3}}{2}$$

$$A_{\square} = 48\sqrt{3}$$

Now summarize what you learned!

---



---



---

### Skillz Review

Important Note:  $(\sin x)(\sin x) = (\sin x)^2 = \sin^2 x$

$(x + 3)(2x - 1)$	$(\sin x + 3)(2\sin x - 1)$	$(\sin x + \cos x)(2\sin x - \cos x)$
$\left(\frac{12}{5}\right)\left(\frac{5}{4}\right) =$	$\left(\frac{12}{\tan x}\right)\left(\frac{\tan x}{4}\right) =$	$\left(\frac{12 \sin x}{\tan x}\right)\left(\frac{\tan x}{4 \cos x}\right) =$