14.1 Practice

Find the next three terms in each sequence. Then, tell if the sequence converges or diverges.

1) 2, 6, 18, 54, 162, ... 486, 1458, 4374

$\cdot 3 \quad \cdot 3 \quad \cdot 3 \quad \cdot 3 \quad \cdot 3$

Diverges

2) −1, 2, 7, 14, 23, ... 34, 47, 62

$+3 \quad +5 \quad +7 \quad +9 \quad +11$

Diverges

3) −3, 15, −75, 375, −1875, ... 9375, −46875, 234375

$\cdot(-5) \cdot(-5) \ (-5)(-5)$

Diverges

4) 1, 1.1, 1.11, 1.111, ... 1.1111, 1.11111, 1.111111

Converges to $1.\overline{1}$ or $\frac{10}{9}$

Find the first four terms in each sequence, given the explicit formula.

5) $a_n = 5^{n-1}$ 1, 5, 25, 125

$a_1 = 5^{1-1} = 5^0 = 1$
$a_2 = 5^{2-1} = 5^1 = 5$
$a_3 = 5^{3-1} = 5^2 = 25$
$a_4 = 5^{4-1} = 5^3 = 125$

6) $a_n = -12 + 30n$ 18, 48, 78, 108

$a_1 = -12 + 30(1) = -12 + 30 = 18$
$a_2 = -12 + 30(2) = -12 + 60 = 48$
$a_3 = -12 + 30(3) = -12 + 90 = 78$
a_4 $= 108$

7) $a_n = n^2 - 1$ 0, 3, 8, 15

$a_1 = (1)^2 - 1 = 1 - 1 = 0$
$a_2 = (2)^2 - 1 = 4 - 1 = 3$
$a_3 = (3)^2 - 1 = 9 - 1 = 8$
$a_4 = (4)^2 - 1 = 16 - 1 = 15$

8) $a_n = \dfrac{8}{n+2}$ $\frac{8}{3}, 2, \frac{8}{5}, \frac{4}{3}$

$a_1 = \frac{8}{1+2} = \frac{8}{3}$
$a_2 = \frac{8}{2+2} = \frac{8}{4} = 2$
$a_3 = \frac{8}{3+2} = \frac{8}{5}$
$a_4 = \frac{8}{4+2} = \frac{8}{6} = \frac{4}{3}$

Find the first four terms in each sequence, given the recursive formula.

9) $a_n = a_{n-1} + \dfrac{3}{2}$ $0, \frac{3}{2}, 3, \frac{9}{2}$

$a_1 = 0$

$a_2 = 0 + \frac{3}{2} = 3/2$
$a_3 = \frac{3}{2} + \frac{3}{2} = \frac{6}{2} = 3$
$a_4 = \frac{6}{2} + \frac{3}{2} = \frac{9}{2}$

10) $a_n = a_{n-1} \cdot -5$ −3, 15, −75, 375

$a_1 = -3$

$a_2 = -3(-5) = 15$

11) $a_n = a_{n-1} \cdot 4$ 3, 12, 48, 192

$a_1 = 3$

$a_2 = 3 \cdot 4 = 12$
$a_3 = 12 \cdot 4 = 48$
$a_4 = 48 \cdot 4 = 192$

12) $a_n = \dfrac{2 + a_{n-1}}{2}$ 10, 6, 4, 3

$a_1 = 10$

$a_2 = \frac{2+10}{2} = \frac{12}{2} = 6$
$a_3 = \frac{2+6}{2} = \frac{8}{2} = 4$
$a_4 = \frac{2+4}{2} = \frac{6}{2} = 3$

Write the explicit formula for each sequence.

13) 4, 20, 100, 500, 2500, ...

$\cdot 5 \quad \cdot 5$

$a_n = 5^n - 5^{n-1}$

14) 29, 20, 11, 2, −7, ...

$-9 \quad -9 \quad -9 \quad -9$

$a_n = 29 - 9n$

Adding consecutive odd integers always implies x^2

15) $\cancel{1}, \frac{3}{2}, \cancel{2}, \frac{5}{2}, \cancel{3}, \ldots$ ($\frac{2}{2}$, $\frac{4}{2}$, $\frac{6}{2}$)

$\boxed{a_n = \frac{n+1}{2}}$

16) $2, 5, 10, 17, 26, \ldots$ ($+3$ $+5$ $+7$ $+9$)

$\boxed{a_n = n^2 + 1}$

Write the recursive formula for each sequence.

17) $3, -6, 12, -24, 48, \ldots$ ((-2) (-2))

$a_n = a_{n-1}(-2)$

$a_1 = 3$

18) $-3, -\frac{3}{4}, -\frac{3}{16}, -\frac{3}{64}, -\frac{3}{256}, \ldots$

$a_n = \frac{a_{n-1}}{4}$

$a_1 = -3$

19) $-4, -8, -16, -32, -64, \ldots$

$a_n = 2 \cdot a_{n-1}$

$a_1 = -4$

20) $3, -\frac{3}{5}, \frac{3}{25}, -\frac{3}{125}, \frac{3}{625}, \ldots$

$a_n = \frac{a_{n-1}}{-5}$

$a_1 = 3$

Evaluate each series.

21) $\sum_{k=1}^{6} (3k^2 - 2)$

$= [3(1)^2-2]+[3(2)^2-2]+[3(3)^2-2]+[3(4)^2-2]+[3(5)^2-2]+[3(6)^2-2]$

$= [3(1)-2]+[3 \cdot 4-2]+[3 \cdot 9-2]+[3(16)-2]+[3(25)-2]+[3(36)-2]$

$= [3-2]+[27-2]+[36-2]+[48-2]+[75-2]+[108-2]$

$= 1 + 25 + 34 + 46 + 73 + 106$

$= 285$

22) $\sum_{a=2}^{8} (20 - a)$

$= [20-2]+[20-3]+[20-4]+[20-5]+[20-6]+[20-7]+[20-8]$

$= 18 + 17 + 16 + 15 + 14 + 13 + 12$

$= 105$

23) $\sum_{k=1}^{6} k(k - 2)$

$= [1(1-2)]+[2(2-2)]+[3(3-2)]+[4(4-2)]+[5(5-2)]+[6(6-2)]$

$= 1(-1) + 2(0) + 3(1) + 4(2) + 5(3) + 6(4)$

$= -1 + 0 + 3 + 8 + 15 + 24$

$= 49$

24) $\sum_{k=4}^{9} k^2 = (4)^2 + (5)^2 + (6)^2 + (7)^2 + (8)^2 + (9)^2$

$= 16 + 25 + 36 + 49 + 64 + 81$

$= 271$

25) $\sum_{m=5}^{11} (40 - m)$

$= (40-5)+(40-6)+(40-7)+(40-8)+(40-9)+(40-10)+(40-11)$

$= 35 + 34 + 33 + 32 + 31 + 30 + 29$

$= 224$

26) $\sum_{k=0}^{4} (3k^2 + 3)$

$= [3(0)^2+3]+[3(1)^2+3]+[3(2)^2+3]+[3(3)^2+3]+[3(4)^2+3]$

$= 0+3 + 3+3 + 3 \cdot 4+3 + 3(9)+3 + 3(16)+3$

$= 3 + 6 + 12+3 + 27+3 + 48+3$

$= 105$

Rewrite each series using sigma notation.

27) $4 + 16 + 64 + 256$

$\sum_{n=1}^{4} 4^n$

28) $1 + 4 + 9 + 16 + 25$

$\sum_{n=1}^{5} n^2$

29) $301 + 302 + 303 + 304 + 305 + 306$

$\sum_{n=301}^{306} n$

30) $601 + 602 + 603 + 604$

$\sum_{n=601}^{604} n$