Pre-Calculus

Write your questions and thoughts here!

14.1 Sequences, Series and Summation (S^3)

Let's look at the following lists of numbers, often called Sequences.

"Repeated addition is multiplication"

1, 3, 5, 7, 9, 11, … (+2 +2 +2 +2 +2 +2)

$a_1 =$ 1

$a_2 =$ 3

$a_{13} =$ 25

Explicit Rule: $a_n =$ $2n-1$

Converge or Diverge? (Diverge)

$\mathbf{0}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \ldots$ ($\frac{0}{1}$)

$a_1 =$ 0 (0)

$a_2 =$ $\frac{1}{2}$ (.5)

$a_{54} =$ $\frac{53}{54}$ (.99)

$a_n =$ $\frac{n-1}{n}$

Converge or Diverge? (Converge to 1)

$-3, 9, -27, 81, \ldots$

$a_1 =$ -3

$a_2 =$ 9

$a_{100} =$ -3^{100}

$a_n = (-3)^n$

Converge or Diverge? (Diverge)

You try!

6, 8, 10, 12, 14 … (+2 +2 +2 +2)

$a_1 =$ 6

$a_2 =$ 8

$a_{19} =$ 42

$a_n =$ $2n+4$

Converge or Diverge? (Diverge)

$1, -1, 1, -1, 1, \ldots$

$a_1 =$ 1

$a_2 =$ -1

$a_{44} =$ -1

$a_n = (-1)^{(n+1)}$

Converge or Diverge? (Diverge)

$\mathbf{-100},\ \mathbf{10},\ \mathbf{-1},\ \mathbf{0.1}, \mathbf{-0.01}, \ldots$ ($\div -10$, $\div -10$)

$a_1 =$ -100

$a_2 =$ 10

$a_9 =$ $\left(\frac{1}{10}\right)^6(-1)^9$

$a_n =$ $(-1)^n 1000\left(\frac{1}{10^n}\right) = \left(\frac{1}{10}\right)^{n-3} (-1)^n$

Converge or Diverge? (Converge to Zero)

Sometimes, we define sequences based on a formula using pervious terms. These formulas are called Recursive.

Example: Find the first four terms of the sequence using the recursive formula that is given:

1. $a_n = 4a_{n-1} - 3$
   $a_1 = 2$
   $a_2 = 4(2) - 3 = 8 - 3 = 5$
   $a_3 = 4(5) - 3 = 20 - 3 = 17$
   $a_4 = 4(17) - 3 = 65 - 3 = 62$

2. $a_n = \frac{1}{2}a_{n-1}$
   $a_1 = 16$
   $a_2 = \frac{1}{2}(16) = 8$
   $a_3 = \frac{1}{2}(8) = 4$
   $a_4 = \frac{1}{2}(4) = 2$

You try!

3. $a_n = n * a_{n-1}$
   $a_1 = 1$
   $a_2 = 2 \cdot (1) = 2$
   $a_3 = 3 \cdot 2 = 6$
   $a_4 = 4 \cdot 6 = 24$

A Series is the sum of all of the terms of a sequence.

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So the sequence *12, 15, 18, 21* would have the corresponding series *of* 12+15+18+21.

To help us write series compactly, we use SIGMA Notation (Summation Notation)

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \cdots + a_{n-1} + a_n$$

Sigma; n ← upper Bound; INDEX; $k=1$ ← Lower Bound

Example: Rewrite each series as a sum. You try!

4. $\sum_{n=0}^{5} 4n$

$= 4(0) + 4(1) + 4(2) + 4(3) + 4(4) + 4(5)$
$= 0 + 4 + 8 + 12 + 16 + 20$

5. $\sum_{p=1}^{4} p^p$

$= (1)^1 + (2)^2 + (3)^3 + (4)^4$
$= 1 + 4 + 27 + 256$

6. $\sum_{m=7}^{10} m(m+3)$

$= 7(7+3) + 8(8+3) + 9(9+3) + 10(10+3)$
$= 7(10) + 8(11) + 9(12) + 10(13)$
$= 70 + 88 + 108 + 130$

Example: Evaluate each series. You try!

7. $\sum_{n=0}^{4} 3^n$

$= 3^0 + 3^1 + 3^2 + 3^3 + 3^4$
$= 1 + 3 + 9 + 27 + 81$
$= 121$

8. $\sum_{h=1}^{4} (-1)^{(h-1)} \frac{1}{h}$

$= (-1)^{(1-1)}\frac{1}{1} + (-1)^{(2-1)}\frac{1}{2} + (-1)^{(3-1)}\frac{1}{3} + (-1)^{(4-1)}\frac{1}{4}$
$= (-1)^0 \cdot 1 + (-1)^1 \frac{1}{2} + (-1)^2 \frac{1}{3} + (-1)^3 \cdot \frac{1}{4}$
$= 1 \cdot 1 + (-1)\frac{1}{2} + 1 \cdot \frac{1}{3} + (-1) \cdot \frac{1}{4}$
$= 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4}$
$= \frac{12}{12} - \frac{6}{12} + \frac{4}{12} - \frac{3}{12}$
$= \frac{7}{12}$

9. $\sum_{k=4}^{7} \frac{k^2}{2}$

$= \frac{(4)^2}{2} + \frac{(5)^2}{2} + \frac{(6)^2}{2} + \frac{(7)^2}{2}$
$= \frac{16 + 25 + 36 + 49}{2}$
$= \frac{126}{2}$
$= 63$

Example: Rewrite the series using sigma notation with *k = 0* and *k = 1*. You try!

10. $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6}$

$\sum_{k=1}^{6} \frac{1}{k}(-1)^{k+1}$

$\sum_{k=0}^{k=5} \frac{1}{k+1}(-1)^k$

11. $1 - \frac{2}{3} + \frac{4}{9} - \frac{8}{27} + \frac{16}{81}$

$\sum_{k=1}^{k=5} \left(\frac{2}{3}\right)^{k-1}(-1)^{k+1}$

$\sum_{k=0}^{k=4} \left(\frac{2}{3}\right)^{k}(-1)^{k}$

Calculator? Mode: Change "Function to Sequence"

Now summarize what you learned!