Series & Sequences – Arithmetic & Geometric Sequences

Hw 14.2

For each sequence, state if it is arithmetic, geometric, or neither. If it is arithmetic, tell the common difference. If it is geometric, tell the common ratio. If it is neither, chill out and move on to the next problem.

1. -1, 6, -36, 216, -1296, ...

Geometric ; Common RATIO: -6

2. 11, -9, -29, -49, -69, ...

Arithmatic ; Common Difference: -20

3. $2, \frac{5}{2}, 3, \frac{7}{2}, 4, \ldots$

$\frac{4}{2}$ $\frac{6}{2}$ $\frac{8}{2}$ Arithmatic ; Common Difference: $\frac{1}{2}$

4. -6, 24, -126, 624, -3126, ...

Neither

5. 32, 36, 40, 44, 48, ...

Arithmatic ; Common Difference: 4

6. 0.4, 2, 10, 50, 250, ...

Geometric ; Common RATIO: 5

7. $a_n = -\frac{19}{24} + \frac{5}{3}n$

Arithmatic ; Common Difference: $\frac{5}{3}$

8. $a_n = 8 + 6n$

Arithmatic ; Common Difference: 6

9. $a_n = 3 \cdot (-6)^{n-1}$

Geometric ; Common RATIO: -6

10. $a_n = \frac{2n}{2n+1}$

Neither

Determine if the sequence is arithmetic. If it is, find the common difference, the term named in the problem, and the explicit formula.

11. 10, 16, 22, 28, ...
Find a_{25}

Arithmatic

CD: 6

$a_n = 10 + 6(n-1)$

$a_{25} = 10 + 6(24) = 10 + 144$

$a_{25} = 154$

12. −31, −33, −35, −37, ...
Find a_{35}

Arithmatic

CD: -2

$a_n = -31 + (-2)(n-1)$

$a_{35} = -31 + (-2)(34) = -31 - 68$

$a_{35} = -99$

13. 1, 2, 6, 24, ...
Find a_{20}

Not Arithmatic

Determine if the sequence is geometric. If it is, find the common ratio, the term named in the problem, and the explicit formula.

14. 1, 4, 16, 64, ...
Find a_9

Geometric

CR: 4

$a_n = (1)4^{(n-1)}$

$a_n = 4^{n-1}$ Explicit

$a_9 = 4^8$

$a_9 = 65,536$

15. −7, −5, −2, 2, ...
Find a_{10}

Not Geometric

16. 1, −2, 4, −8, ...
Find a_{10}

Geometric

CR: -2

$a_n = (1)(-2)^{n-1}$

$a_n = (-2)^{n-1}$ Explicit

$a_{10} = (-2)^9$

$a_{10} = -512$

For numbers 16 – 20, find the sum of the first ***n*** terms indicated in part (a). Then, for part (b), find ***n*** for the given sum S_n.

17. $1 + 4 + 16 + 64 + \ldots$ Geo $r = 4$

a. *Sum of the first 14* terms?
b. For which term would $S_n = 341$?

a $S_n = a_1\left(\frac{1-r^n}{1-r}\right)$ Explicit
$S_n = 1\left(\frac{1-4^n}{1-4}\right)$
$S_{14} = \left(\frac{1-4^{14}}{1-4}\right)$
$S_{14} = 89,478,485$

b. $341 = 1\cdot\left(\frac{1-4^n}{1-4}\right)$
$341 = \frac{1-4^n}{-3}$
$-1023 = 1-4^n$
$-1024 = -4^n$
$\log 1024 = \log 4^n$
$\log 1024 = n\log 4$
$\frac{\log 1024}{\log 4} = n$
$n = 5$

I realize this is logic and not math

18. $50 + 42 + 34 + 26 + \ldots$ Arith $cd = -8$

a. *Sum of the first 40* terms?
b. For which term would $S_n = 182$?

a. $S_n = \frac{n(a_1+a_n)}{2}$
$S_{40} = \frac{40(50+-262)}{2}$
$S_{40} = -4240$

$a_n = a_1 + d(n-1)$
$a_n = 50 + -8(n-1)$ Explicit
$a_{40} = 50 + -8(39)$
$a_{40} = -262$

b. $182 = \frac{n[50+(50+-8(n-1))]}{2}$
$364 = n[50+50-8n+8]$
$364 = n[108-8n]$
$364 = -8n^2+108n$
$8n^2-108n+364=0$

$4(2n^2-27n+91)=0$
$4[2n^2-14n+73n+91]=0$
$4[2n(n-7)-13(n-7)]=0$
$4(n-7)(2n-13)=0$
$n-7=0$ $\quad n=7$
$2n-13=0$ $\quad 2n=13$ $\quad n=13/2$ ← can't be fraction

19. $7 + (-21) + 63 + (-189) + \ldots$ Geo $r = -3$

a. *Sum of the first 84* terms?
b. For which term would $S_n = 3829$?

a. $S_n = a_1\left(\frac{1-r^n}{1-r}\right)$ Explicit
$S_n = 7\left(\frac{1-(-3)^n}{1-(-3)}\right)$
$S_{84} = 7\left(\frac{1-(-3)^{84}}{1-(-3)}\right)$
$S_{84} = -2.09519\cdot 10^{40}$

b. $3829 = 7\left(\frac{1-(-3)^n}{1-(-3)}\right)$
$3829 = 7\left(\frac{1-(-3)^n}{4}\right)$
$547 = \frac{1-(-3)^n}{4}$
$2188 = 1-(-3)^n$
$2187 = -(-3)^n$

Since the left side is +, the right side is too.
$\log 2187 = \log 3^n$

$2187 = (3)^n$
$\log(2187) = \log(3)^n$
$\log(2187) = n\log(3)$
$\frac{\log(2187)}{\log(3)} = n$
$7 = n$

20. $2 + 16 + 30 + 44 + 58 + \ldots$ Arith $cd = 14$

a. *Sum of the first 24* terms?
b. For which term would $S_n = 2178$

a. $S_n = \frac{n(a_1+a_n)}{2}$
$S_{24} = \frac{24(2+324)}{2}$
$S_{24} = 3912$

$a_n = a_1 + d(n-1)$
$a_n = 2 + 14(n-1)$ Explicit
$a_{24} = 2 + 14(23)$
$a_{24} = 324$

b. $2178 = \frac{n(2+(2+14(n-1)))}{2}$
$4356 = n(2+2+14n-14)$
$4356 = n(14n-10)$
$0 = 14n^2-10n-4356$
$0 = 2(7n^2-5n-2178)$
$0 = 2[7n^2-126n+121n-2178]$
$0 = 2[7n(n-18)+121(n-18)]$
$0 = 2(n-18)(7n+121)$
$n-18=0$ ↳ FRACTION
$n = 18$

21. $1 + 9 + 81 + 729 + \ldots$ Geo $r=3$

a. *Sum of the first 10* terms?

b. For which term would $S_n = 820$?

a. $S_n = a_1\left(\frac{1-r^n}{1-r}\right)$

$S_n = 1\left(\frac{1-9^n}{1-9}\right)$ Explicit

$S_{10} = 1\left(\frac{1-9^{10}}{1-9}\right)$

$S_{10} = 435,848,050$

b. $820 = 1\cdot\left(\frac{1-9^n}{1-9}\right)$

$820 = \frac{1-9^n}{-9}$

$-6560 = 1-9^n$

$-6561 = -9^n$

$6561 = 9^n$

$\log 6561 = \log 9^n$

$\log 6561 = n\log 9$

$\frac{\log 6561}{\log 9} = n$

$4 = n$

22. $3 + 8 + 13 + 18 + 23 + \ldots$ Arith CD $=5$

a. *Sum of the first 20* terms?

b. For which term would $S_n = 366$?

a. $S_n = \frac{n(a_1+a_n)}{2}$

$S_{20} = \frac{20(3+285)}{2}$

$S_{20} = 1010$

$a_n = a_1 + d(n-1)$

$a_n = 3 + 5(n-1)$ Explicit

$a_n = 3+5(19)$

$a_n = 285$

b. $S_n = \frac{n(a_1+a_n)}{2}$

$366 = \frac{n(3+(3+5(n-1)))}{2}$

$732 = n(3+3+5n-5)$

$732 = n(5n+1)$

$0 = 5n^2 + n - 732$

$0 = 5n^2 - 60n + 61n - 732$

$0 = 5n(n-12) + 61(n-12)$

$0 = (n-12)(5n+61)$

$n-12=0$ ← FRACTION

$n=12$

Evaluate each series.

23. $\sum_{i=2}^{7} i+2$ $= (2+2)+(3+2)+(4+2)+(5+2)+(6+2)+(7+2)$

$= 4 + 5 + 6 + 7 + 8 + 9$

$= 39$

24. $\sum_{j=1}^{3} j^j$ $= 1^1 + 2^2 + 3^3$

$= 1 + 4 + 27$

$= 32$

25. $\sum_{k=3}^{5} t^k$ $= t^3 + t^4 + t^5$

Write each series in sigma notation.

26. $16 + 25 + 36 + 49 + 64$

$\sum_{i=4}^{8} i^2$

27. $2 + 4 + 8 + 16 + 32$

$\sum_{i=1}^{5} 2^i$

28. $501 + 502 + 503 + 504$

$\sum_{i=1}^{4} 500 + i$

Skills Review! Write the equation of a line with the given slope that passes through the given point.

In slope-intercept form: $y = mx + b$		In point-slope form: $y - y_1 = m(x - x_1)$	
1. slope = -3; through (-1,3)	2. slope = 0; through (-2,3)	3. slope = 3; through (1,-3)	4. slope = $-\frac{3}{5}$; through (0,0)
$y - y_1 = m(x - x_1)$ $y - (3) = -3(x - (-1))$ $y - 3 = -3x - 3$ $y = -3x$	horizontal $y = -2$	$y - y_1 = m(x - x_1)$ $y - (-3) = 3(x - (1))$ $y + 3 = 3x - 3$ $y = 3x - 6$	y-int $y = -\frac{3}{5}x$