Series & Sequences – Arithmetic & Geometric Sequences
Notes 14.2

Nth term of an Arithmetic Sequence

The n^{th} term of an arithmetic sequence with first term a_1 and common difference *d* is given by:

$$a_n = a_1 + d(n-1)$$

An **arithmetic sequence** is one in which the same number is added or subtracted from each term to get the next term in the **sequence**. The number you add or subtract is called the ***common difference***.

Handy rules involving an arithmetic sequence:

$$a_1 = a_1$$
$$a_2 = a_1 + d$$
$$a_3 = a_1 + 2d$$
$$a_4 = a_1 + 3d$$
$$a_5 = a_1 + 4d$$
$$\vdots$$

Are the following arithmetic sequences?

1. 3, 8, 13, 18, 23, 28...
   +5 +5 +5 +5 +5 Yes
2. -2, -12, -22, -32, ...
   -10 -10 -10 Yes
3. 2, 4, 8, 16, 32, ...
   +2 +4 +8 +16 no
4. 14, 14.5, 15, 15.5, 16...
   +.5 +.5 +.5 +.5 Yes

Given two terms in an arithmetic sequence, find the common difference, the 52nd term, and the explicit formula.

5. $a_{20} = 70$

$d = \frac{a_{33} - a_{20}}{33 - 20}$

$d = \frac{26}{13}$

$d = 2$

$a_{20} = a_1 + (20)2$

$70 = a_1 + 40$

$30 = a_1$

6. $a_{33} = 96$

$a_n = 30 + 2(n-1)$ Explicit

$a_{52} = 30 + (51)2$

$a_{52} = 30 + 102$

$a_{52} = 132$

Sum of a Finite Arithmetic Sequence

The sum of the first *n* terms of an arithmetic sequence is given by:

$$S_n = \frac{n(a_1 + a_n)}{2}$$

Find the sum of the first 20 terms of the arithmetic series:

7. 2 + 6 + 10 + 14 + 18 + . . .

$a_n = a_1 + d(n-1)$

$a_n = 2 + 4(n-1)$ Explicit

$a_{20} = 2 + (19)4$

$a_{20} = 2 + 76$

$a_{20} = 78$

$S_{20} = \frac{20(2+78)}{2}$

$= 10(80)$

$S_{20} = 800$

8. Suppose the sum of the series has a sum of 2178. Find n such that *Sn* = 2178.

$a_n = 2 + 4(n-1)$ Explicit

$= 2 + 4n - 4$

$a_n = 4n - 2$

$S_n = \frac{n(a_1 + a_n)}{2}$

$2178 = \frac{n(\cancel{2} + 4n - \cancel{2})}{2}$

$2178 = \frac{n(4n)}{2}$

$2178 = 2n^2$

$1089 = n^2$

$\pm 33 = n$

So $n = 33$

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Nth term of a Geometric Sequence

The n^{th} term of a geometric sequence with first term a_1 and common ratio r is given by:

$$a_n = a_1 \cdot r^{n-1}$$

A **geometric sequence** is one in which the same number is multiplied or divided by each term to get the next term in the **sequence**. The number you multiply or divide by is called the ***common ratio***, usually denoted by ***r***.

Determine if the following sequences are arithmetic, geometric or neither.

9. 1, 2, 6, 24, 120, … Neither

$\cdot 2 \cdot 3 \cdot 4 \cdot 5$

10. 81, 27, 9, 3, 1, …

$\div 3 \div 3 \div 3 \div 3 \div 3$ Geometric

11. 5, 10, 15, 20, 25, …

$+5 +5 +5 +5$ Arithmatic

The third term of a geometric series equals 64 while the common ratio is 2.

12. Write a rule for the nth term.

$a_n = a_1 r^{n-1}$

$a_3 = a_1 (2)^{3-1}$

$64 = a_1 2^2$

$64 = a_1 4$

$16 = a_1$

$a_n = 16 \cdot 2^{n-1}$

Explicit

13. Find the 9th term

$a_9 = 16 \cdot 2^8$

$a_9 = 16 \cdot 256$

$\boxed{a_9 = 4096}$

Sum of a Finite Geometric Series

The sum of the first n terms of an geometric sequence with r as the common ratio is given by:

CR $S_n = a_1 \left(\dfrac{1 - r^n}{1 - r}\right)$

14. Find the sum of the first 10 terms of the series

1 + 5 + 25 + 125 + … CR=5

$S_{10} = 1\left(\dfrac{1-5^{10}}{1-5}\right)$

$= \dfrac{1-9765625}{-4}$

$= \dfrac{-9765624}{-4}$

$S_{10} = 2441406$

15. For which term would $S_n = 3906$?

$S_n = a_1\left(\dfrac{1-r^n}{1-r}\right)$

$S_n = 1 \cdot \left(\dfrac{1-5^n}{1-5}\right)$ Explicit

$3906 = \left(\dfrac{1-5^n}{1-5}\right)$

$3906 = \dfrac{1-5^n}{-4}$

$-15{,}624 = 1 - 5^n$

$-15{,}625 = -5^n$

$15{,}625 = 5^n$

$\log 15{,}625 = \log 5^n$

$\log 15{,}625 = n \log 5$

$\dfrac{\log 15{,}625}{\log 5} = n$

$6 = n$

The n^{th} term is 6