

Write your questions and thoughts here!

## The Binomial Theorem

Expansion, the boring and tedious way:

$$\begin{aligned}
 (a + b)^0 &= 1 \\
 (a + b)^1 &= a + b \\
 (a + b)^2 &= a^2 + 2ab + b^2 \\
 (a + b)^3 &= a^3 + 3a^2b + 3ab^2 + b^3 \\
 (a + b)^4 &= \\
 (a + b)^5 &=
 \end{aligned}$$

The AMAZING Binomial Theorem:

$$(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$$

where  $\binom{n}{k}$  is said "n choose k" and represents the number of ways to select k things from n.

where  $\binom{n}{k} = {}_n C_k = \frac{n!}{k!(n-k)!}$  and  $n! = n(n-1)(n-2)\dots(2)1$

EXAMPLES:

Evaluate:  $\binom{5}{3} = 5C_3 = \frac{5!}{3!2!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 2 \cdot 1} = 10$        $\binom{7}{2} = 7C_2 = \frac{7!}{2!5!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 21$        $\binom{7}{5} = 7C_5 = \frac{7!}{5!2!} = \frac{7 \cdot 6}{2 \cdot 1} = 21$

Binomial Expansion, using the amazing Binomial Theorem:  $(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$

$$\begin{aligned}
 (a + b)^3 &= \binom{3}{0} a^{3-0} b^0 + \binom{3}{1} a^{3-1} b^1 + \binom{3}{2} a^{3-2} b^2 + \binom{3}{3} a^{3-3} b^3 \\
 &= a^3 + 3a^2b + 3ab^2 + b^3
 \end{aligned}$$

$$\begin{aligned}
 (2x - 3y)^3 &= \binom{3}{0} (2x)^3 (-3y)^0 + \binom{3}{1} (2x)^2 (-3y)^1 + \binom{3}{2} (2x)^1 (-3y)^2 + \binom{3}{3} (2x)^0 (-3y)^3 \\
 &= 1 \cdot 8x^3 \cdot 1 + 3(4x^2)(-3y) + 3(2x)(9y^2) + 1 \cdot 1 \cdot (-27y^3) \\
 &= 8x^3 - 36x^2y + 54xy^2 - 27y^3
 \end{aligned}$$

You try! Expand completely.

$$\begin{aligned}
 (5x + 3y)^4 &= \binom{4}{0} (5x)^4 (3y)^0 + \binom{4}{1} (5x)^3 (3y)^1 + \binom{4}{2} (5x)^2 (3y)^2 + \binom{4}{3} (5x)^1 (3y)^3 + \binom{4}{4} (5x)^0 (3y)^4 \\
 &= 1 \cdot 625x^4 \cdot 1 + 4 \cdot 125x^3 \cdot 3y + 6 \cdot 25x^2 \cdot 9y^2 + 4 \cdot 5x \cdot 27y^3 + 1 \cdot 1 \cdot 81y^4 \\
 &= 625x^4 + 1500x^3y + 1350x^2y^2 + 540xy^3 + 81y^4
 \end{aligned}$$

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SIDE WORK

$$\begin{aligned}
 \binom{3}{0} &= \frac{3!}{0!3!} = 1 \\
 \binom{3}{1} &= \frac{3!}{1!2!} = 3 \\
 \binom{3}{2} &= \frac{3!}{2!1!} = 3 \\
 \binom{3}{3} &= \frac{3!}{3!0!} = 1
 \end{aligned}$$

$$\begin{aligned}
 \binom{4}{0} &= \frac{4!}{0!4!} = 1 \\
 \binom{4}{1} &= \frac{4!}{1!3!} = 4 \\
 \binom{4}{2} &= \frac{4!}{2!2!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1} = 6 \\
 \binom{4}{3} &= \frac{4!}{3!1!} = 4 \\
 \binom{4}{4} &= \frac{4!}{4!0!} = 1
 \end{aligned}$$

# 14.3 The Binomial Theorem

Write your questions and thoughts here!

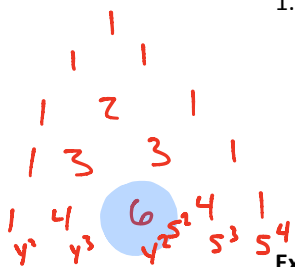
Pascal's Triangle	
$(a + b)^0 =$ $(a + b)^1 =$ $(a + b)^2 =$ $(a + b)^3 =$ $(a + b)^4 =$ $(a + b)^5 =$ $\vdots$	$1$ $1a + 1b$ $1a^2 + 2ab + 1b^2$ $1a^3 + 3a^2b + 3ab^2 + 1b^3$ $1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$ $1a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + 1b^5$ $\vdots$
Looking at just the coefficients:	$1$ $1 \ 1$ $1 \ 2 \ 1$ $1 \ 3 \ 3 \ 1$ $1 \ 4 \ 6 \ 4 \ 1$ $1 \ 5 \ 10 \ 10 \ 5 \ 1$ $\vdots$
Which happens to be the same as:	${}^0C_0$ ${}^1C_0 \ 1C_1$ ${}^2C_0 \ 2C_1 \ 2C_2$ ${}^3C_0 \ 3C_1 \ 3C_2 \ 3C_3$ ${}^4C_0 \ 4C_1 \ 4C_2 \ 4C_3 \ 4C_4$ ${}^5C_0 \ 5C_1 \ 5C_2 \ 5C_3 \ 5C_4 \ 5C_5$ $\vdots$

**EXAMPLES:**

Find each term described.

1. The third term in the expansion  $(y + 5)^4$

2. The third term in the expansion  $(2x - 3)^6$

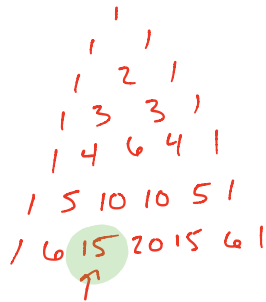


$$6y^2 5^2 = 6y^2 25 = 150y^2$$

Expand completely.

3.  $(2 + 3y)^4$

$$\begin{aligned}
 &= 1 \cdot (2)^4 (3y)^0 + 4(2)^3 (3y)^1 + 6(2)^2 (3y)^2 + 4(2)^1 (3y)^3 + 1(2)^0 (3y)^4 \\
 &= 16 \cdot 1 + 4 \cdot 8 \cdot 3y + 6 \cdot 4 \cdot 9y^2 + 4 \cdot 2 \cdot 27y^3 + 1 \cdot 1 \cdot 81y^4 \\
 &= 16 + 96y + 216y^2 + 216y^3 + 81y^4
 \end{aligned}$$



$$\begin{aligned}
 &15(2x)^4(-3)^2 \\
 &= 15 \cdot 16x^4 \cdot 9 \\
 &= 2160x^4
 \end{aligned}$$

Now summarize what you learned!

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