Write your questions and thoughts here!

The Binomial Theorem

Expansion, the boring and tedious way:

$(a+b)^0 = 1$
$(a+b)^1 = a+b$
$(a+b)^2 = a^2+2ab+b^2$
$(a+b)^3 = a^b+3a^2b+3ab^2+b^2$
$(a+b)^4 =$
$(a+b)^5 =$

The AMAZING Binomial Theorem:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

where $\binom{n}{k}$ is said "n choose k" and represents the number of ways to select k things from n.

where $\binom{n}{k} = {}_nC_k = \frac{n!}{k!(n-k)!}$ and $n! = n(n-1)(n-2)\ldots(2)1$

MATH

PBR

7 nCr 5

EXAMPLES:

Evaluate: $\binom{5}{3} = {}_5C_3 = \frac{5!}{3!\,2!} = \frac{5\cdot \cancel{4}^{2}\cdot \cancel{3\cdot 2\cdot 1}}{\cancel{3\cdot 2\cdot 1}\cdot \cancel{2}\cdot 1} = 10$

$\binom{7}{2}$: ${}_7C_2 = \frac{7!}{2!\,5!} = \frac{7\cdot \cancel{6}^{3}\cdot \cancel{5\cdot 4\cdot 3\cdot 2\cdot 1}}{\cancel{2}\cdot 1\cdot \cancel{5\cdot 4\cdot 3\cdot 2\cdot 1}} = 21$

$\binom{7}{5} = {}_7C_5 = \frac{7!}{5!\,2!} = \frac{7\cdot 6}{2} = 21$

Binomial Expansion, using the amazing Binomial Theorem: $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$

SIDEWORK

$\binom{3}{0} = \frac{3!}{0!\,3!} = 1$

$\binom{3}{1} = \frac{3!}{1!\,2!} = 3$

$\binom{3}{2} = \frac{3!}{2!\,1!} = 3$

$\binom{3}{3} = \frac{3!}{3!\,0!} = 1$

$(a+b)^3 = \binom{3}{0}a^{3-0}b^0 + \binom{3}{1}a^{3-1}b^1 + \binom{3}{2}a^{3-2}b^2 + \binom{3}{3}a^{3-3}b^3$
$= a^3 + 3a^2b + 3ab^2 + b^3$

$(2x-3y)^3 = \binom{3}{0}(2x)^3(-3y)^0 + \binom{3}{1}(2x)^2(-3y)^1 + \binom{3}{2}(2x)^1(-3y)^2 + \binom{3}{3}(2x)^0(-3y)^3$
$= 1\cdot 8x^3\cdot 1 + 3(4x^2)(-3y) + 3(2x)(9y^2) + 1\cdot 1\cdot(-27y^3)$
$= 8x^3 - 36x^2 + 54xy^2 - 27y^3$

You try! Expand completely.

$(5x+3y)^4$
$= \binom{4}{0}(5x)^4(3y)^0 + \binom{4}{1}(5x)^3(3y)^1 + \binom{4}{2}(5x)^2(3y)^2 + \binom{4}{3}(5x)^1(3y)^3 + \binom{4}{4}(5x)^0(3y)^4$
$= 1\cdot 625x^4\cdot 1 + 4\cdot 125x^3\cdot 3y + 6\cdot 25x^2\cdot 9y^2 + 4\cdot 5x\cdot 27y^3 + 1\cdot 1\cdot 81y^4$
$= 625x^4 + 1500x^3y + 1350x^2y^2 + 540xy^3 + 81y^4$

$\binom{4}{0} = \frac{4!}{0!\,4!} = 1$

$\binom{4}{1} = \frac{4!}{1!\,3!} = 4$

$\binom{4}{2} = \frac{4!}{2!\,2!} = \frac{\cancel{4}^{2}\cdot 3\cdot \cancel{2}\cdot 1}{\cancel{2}\cdot 1\cdot \cancel{2}\cdot 1} = 6$

$\binom{4}{3} = \frac{4!}{3!\,1!} = 4$

$\binom{4}{4} = \frac{4!}{4!\,0!} = 1$

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Pascal's Triangle	
$(a+b)^0$ = $(a+b)^1$ = $(a+b)^2$ = $(a+b)^3$ = $(a+b)^4$ = $(a+b)^5$ = ⋮	1 $1a+1b$ $1a^2+2ab+1b^2$ $1a^3+3a^2b+3ab^2+1b^2$ $1a^4+4a^3b+6a^2b^2+4ab^3+1b^4$ $1a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+1b^5$ ⋮
Looking at just the coefficients:	1 1 1 1 2 1 1 3 3 1 1 4 6 4 1 1 5 10 10 5 1 ⋮
Which happens to be the same as:	${}_0C_0$ ${}_1C_0$ ${}_1C_1$ ${}_2C_0$ ${}_2C_1$ ${}_2C_2$ ${}_3C_0$ ${}_3C_1$ ${}_3C_2$ ${}_3C_3$ ${}_4C_0$ ${}_4C_1$ ${}_4C_2$ ${}_4C_3$ ${}_4C_4$ ${}_5C_0$ ${}_5C_1$ ${}_5C_2$ ${}_5C_3$ ${}_5C_4$ ${}_5C_5$ ⋮

EXAMPLES:

Find each term described.

1. The third term in the expansion $(y + 5)^4$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
y^4 y^3 $y^2 5^2$ 5^3 5^4

$6y^2 5^2$

$= 6y^2 25$

$= 150y^2$

2. The third term in the expansion $(2x - 3)^6$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

$15(2x)^4(-3)^2$

$= 15 \cdot 16x^4 \cdot 9$

$= 2160x^4$

Expand completely.

3. $(2 + 3y)^4$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

$= 1\cdot(2)^4(3y)^0 + 4(2)^3(3y)^1 + 6(2)^2(3y)^2 + 4(2)^1(3y)^3 + 1(2)^0(3y)^4$

$= 16\cdot 1 + 4\cdot 8\cdot 3y + 6\cdot 4\cdot 9y^2 + 4\cdot 2(27y^3) + 1\cdot 1\cdot 81y^4$

$= 16 + 96y + 216y^2 + 216y^3 + 81y^4$

Now summarize what you learned!