Name: ______________________________ Date:______________ Period: _____

15 REVIEW – Intro to Calculus

Pre-Calculus

Evaluate each limit.

1. $\lim\limits_{x\to -1}(4x^2-2x+1)$

$= 4(-1)^2-2(-1)+1$

$= 4(1)+2+1$

$= 7$

2. $\lim\limits_{x\to 3}\sqrt{2x-2}$

$= \sqrt{2(3)-2}$

$= \sqrt{6-2}$

$= \sqrt{4}$

$= 2$

3. $\lim\limits_{x\to 2} 7$

$= 7$

4. $\lim\limits_{x\to 1}\dfrac{\sqrt{x+2}+\sqrt{3}}{2x}$

$= \dfrac{\sqrt{(1)+2}+\sqrt{3}}{2(1)}$

$= \dfrac{\sqrt{3}+\sqrt{3}}{2}$

$= \dfrac{2\sqrt{3}}{2}$

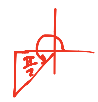
$= \sqrt{3}$

5. $\lim\limits_{x\to \frac{\pi}{2}}\sin(3x)$

$= \sin\left[3\left(\frac{\pi}{2}\right)\right]$

$= \sin\frac{3\pi}{2}$

$= -\dfrac{\sqrt{2}}{2}$



6. $\lim\limits_{x\to 2}\dfrac{x^2-4x}{x-2}$

$= \lim\limits_{x\to 2}\dfrac{x(x-4)}{x-2}$

$=$ dne

7. $\lim\limits_{x\to 5}\dfrac{x^2+x-30}{x-5}$

$= \lim\limits_{x\to 5}\dfrac{(x-5)(x+6)}{x-5}$

$= \lim\limits_{x\to 5}(x+6)$

$= 5+6$

$= 11$

8. $\lim\limits_{x\to 0}\dfrac{3x^3-5x^2+13x}{x^2-7x}$

$= \lim\limits_{x\to 0}\dfrac{\cancel{x}(3x^2-5x+13)}{\cancel{x}(x-7)}$

$= \dfrac{3(0)^2-5(0)+13}{0-7}$

$= \dfrac{13}{-7}$

9. $\lim\limits_{x\to 3}\dfrac{x^2-2x-3}{3-x}$

$= \lim\limits_{x\to 3}\dfrac{(x-3)(x+1)}{-1(x-3)}$

$= \lim\limits_{x\to 3}\dfrac{x+1}{-1}$

$= \dfrac{3+1}{-1}$

$= \dfrac{4}{-1}$

$= -4$

10. $\lim\limits_{x\to 0}\dfrac{\left(\sqrt{x+10}-\sqrt{10}\right)}{x}\cdot\dfrac{\left(\sqrt{x+10}+\sqrt{10}\right)}{\left(\sqrt{x+10}+\sqrt{10}\right)}$

$= \lim\limits_{x\to 0}\dfrac{x+10-10}{x\left(\sqrt{x+10}+\sqrt{10}\right)}$

$= \lim\limits_{x\to 0}\dfrac{\cancel{x}}{\cancel{x}\left(\sqrt{x+10}+\sqrt{10}\right)}$

$= \lim\limits_{x\to 0}\dfrac{1}{\sqrt{x+10}+\sqrt{10}}$

$= \dfrac{1}{\sqrt{0+10}+\sqrt{10}}$

$= \dfrac{1}{2\sqrt{10}}$

11. $\lim\limits_{x\to 0}\dfrac{\left(\sqrt{x+16}-4\right)}{x}\cdot\dfrac{\left(\sqrt{x+16}+4\right)}{\left(\sqrt{x+16}+4\right)}$

$= \lim\limits_{x\to 0}\dfrac{x+16-16}{x\left(\sqrt{x+16}+4\right)}$

$= \lim\limits_{x\to 0}\dfrac{\cancel{x}}{\cancel{x}\left(\sqrt{x+16}+4\right)}$

$= \lim\limits_{x\to 0}\dfrac{1}{\sqrt{x+16}+4}$

$= \dfrac{1}{\sqrt{0+16}+4}$

$= \dfrac{1}{\sqrt{16}+4} = \dfrac{1}{4+4} = \dfrac{1}{8}$

12. $\lim\limits_{x\to 0}\dfrac{\frac{5}{5}\cdot\frac{1}{x+5}-\frac{1}{5}\cdot\frac{(x+5)}{(x+5)}}{x}$

$= \lim\limits_{x\to 0}\dfrac{\frac{5}{5(x+5)}-\frac{x+5}{5(x+5)}}{x}$

$= \lim\limits_{x\to 0}\dfrac{\frac{-\cancel{x}}{5(x+5)}}{\cancel{x}}\cdot\dfrac{\frac{1}{\cancel{x}}}{\frac{1}{\cancel{x}}} = \lim\limits_{x\to 0}\dfrac{-1}{5(x+5)}$

$= \dfrac{-1}{5(0+5)} = \dfrac{-1}{5(5)} = \dfrac{-1}{25}$

Find the derivative ***using limits***. (SHOW WORK!)

13. $y = 4-3x$

$y' = \lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0}\dfrac{[4-3(x+h)]-[4-3x]}{h}$

$= \lim\limits_{h\to 0}\dfrac{\cancel{4}-\cancel{3x}-3h-\cancel{4}+\cancel{3x}}{h} = \lim\limits_{h\to 0}\dfrac{-3h}{h}$

$= \lim\limits_{h\to 0} -3$

$= -3$

14. $y = 4x^2-6x+3$

$y' = \lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0}\dfrac{[4(x+h)^2-6(x+h)+3]-[4x^2-6x+3]}{h}$

$= \lim\limits_{h\to 0}\dfrac{\cancel{4x^2}+8hx+4h^2-\cancel{6x}-6h+\cancel{3}-\cancel{4x^2}+\cancel{6x}-\cancel{3}}{h}$

$= \lim\limits_{h\to 0}\dfrac{h(8x+4h-6)}{h}$

$= \lim\limits_{h\to 0}(8x+4h-6)$

$= 8x+4(0)-6$

$= 8x-6$

15. $f(x) = \sqrt{6x-5}$

$f'(x) = \lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0}\dfrac{\sqrt{6(x+h)-5}-\sqrt{6x-5}}{h}$

$= \lim\limits_{h\to 0}\dfrac{\sqrt{6x+6h-5}-\sqrt{6x-5}}{h}\cdot\dfrac{\left(\sqrt{6x+6h-5}+\sqrt{6x-5}\right)}{\left(\sqrt{6x+6h-5}+\sqrt{6x-5}\right)}$

$= \lim\limits_{h\to 0}\dfrac{(\cancel{6x}+6h\cancel{-5})-(\cancel{6x}\cancel{-5})}{h\left(\sqrt{6x+6h-5}+\sqrt{6x-5}\right)}$

$= \lim\limits_{h\to 0}\dfrac{6\cancel{h}}{\cancel{h}\left(\sqrt{6x+6h-5}+\sqrt{6x-5}\right)}$

$= \dfrac{6}{\sqrt{6x+6(0)-5}+\sqrt{6x-5}}$

$= \dfrac{6}{\sqrt{6x-5}+\sqrt{6x-5}}$

$= \dfrac{6}{2\sqrt{6x-5}}$

$= \dfrac{3}{\sqrt{6x-5}}$

For each problem, create an equation of the tangent line of f at the given point. Answer can be in point-slope form OR slope-intercept.

16. $f(-5) = 9$ and $f'(-5) = -4$

$(-5, 9)$ $\quad m = -4$

$y - 9 = -4[x - (-5)]$

17. $f(2) = -5$ and $f'(2) = 3$

$(2, -5)$ $\quad m = 3$

$y - (-5) = 3[x - 2]$

18. If $f(x) = \sin 4x$ and its derivative is $f'(x) = 4\cos 4x$, find an equation of the tangent line at $x = \frac{\pi}{4}$.

Point @ $x = \frac{\pi}{4}$

$f(\frac{\pi}{4}) = \sin 4(\frac{\pi}{4})$
$= \sin \pi$
$f(\frac{\pi}{4}) = 0 \Rightarrow (\frac{\pi}{4}, 0)$

Slope @ $x = \frac{\pi}{4}$

$f'(\frac{\pi}{4}) = 4\cos 4(\frac{\pi}{4})$
$= 4\cos \pi$
$= 4 \cdot (-1)$
$f'(\frac{\pi}{4}) = -4 \Rightarrow m = -4$

Point-Slope

$y - y_1 = m(x - x_1)$
$y - 0 = -4(x - \frac{\pi}{4})$
$y = -4x + \pi$

Find the derivative of each expression and simplify.

19. $f(x) = 6e^5$

$f'(x) = 0$

20. $h(x) = \frac{x}{5} = \frac{1}{5}x$

$h'(x) = \frac{1}{5}$

21. $w(t) = 4$

$w(t) = 0$

22. $y = 5x^2 + 10x - 8$

$y' = 10x + 10$

23. $y = \frac{9}{x} = 9x^{-1}$

$y' = -9x^{-2}$

24. $f(x) = \frac{5}{x^2} = 5x^{-2}$

$f'(x) = -10x^{-3}$

25. $6\sqrt{x} = 6x^{\frac{1}{2}}$

$(6\sqrt{x})' = 3x^{-\frac{1}{2}}$

26. $\sqrt[7]{x^8} = x^{7/8}$

$(\sqrt[7]{x^8})' = \frac{7}{8}x^{-1/8}$

27. $\sqrt{x}(\sqrt[5]{x} - \sqrt[3]{x})$ Step 1 $= x^{\frac{1}{2}}(x^{\frac{1}{5}} - x^{\frac{1}{3}})$

$= x^{\frac{1}{2}+\frac{1}{5}} - x^{\frac{1}{2}+\frac{1}{3}}$
$= x^{\frac{5}{10}+\frac{2}{10}} - x^{\frac{3}{6}+\frac{2}{6}}$
$= x^{7/10} - x^{5/6}$

Step 2 $(x^{7/10} - x^{5/6})'$
$= \frac{7}{10}x^{3/10} - \frac{5}{6}x^{1/6}$

28. If $f(x) = \frac{1}{\sqrt[3]{x}}$ find the value of the derivative at $x = 8$.

$f(x) = x^{-\frac{1}{3}}$

$f'(x) = -\frac{1}{3}x^{-4/3} = \frac{-1}{3\sqrt[3]{x^4}}$

$f'(8) = \frac{-1}{3(\sqrt[3]{8})^4}$
$= \frac{-1}{3(2)^4}$
$= \frac{-1}{3(16)}$
$f'(8) = -\frac{1}{48}$

29. Determine the x-value(s) at which $y = \frac{1}{3}x^3 - \frac{3}{2}x^2$ has a horizontal tangent line.

$y' = x^2 - 3x$

@ horizontal, $m = 0$
$0 = x^2 - 3x$
$0 = x(x - 3)$
$0 = x$; $0 = x - 3$
$3 = x$

Find the equation of a tangent line of each function at the indicated point.

30. $f(x) = 3\sqrt{x} - x^2; \quad x = 4$

Point @ $x = 4$

$f(4) = 3\sqrt{4} - (4)^2$
$= 3 \cdot 2 - 2$
$= 6 - 2$
$f(4) = 4$
$(4, 4)$

Slope @ $x = 4$

$f'(x) = \frac{3}{2}x^{-\frac{1}{2}} - 2x$
$f'(4) = \frac{3}{2\sqrt{4}} - 2(4)$
$= \frac{3}{2 \cdot 2} - 8$
$= \frac{3}{4} - \frac{32}{4}$
$f'(4) = -\frac{29}{4}$
$m = -\frac{29}{4}$

Point-Slope

$y - y_1 = m(x - x_1)$
$y - 4 = -\frac{29}{4}(x - 4)$
$y - 4 = -\frac{29}{4} + 29$
$y = -\frac{29}{4} + 33$

31. $f(x) = 3x^2 + 2x; \; x = -2$

Point @ $x = -2$

$f(-2) = 3(-2) + 2(-2)$
$= -6 - 4$
$f(-2) = -10$
$(-2, -10)$

Slope @ $x = -2$

$f'(x) = 6x + 2$
$f'(-2) = 6(-2) + 2$
$= -12 + 2$
$f'(-2) = -10$
$m = -10$

Point-Slope

$y - y_1 = m(x - x_1)$
$y - (-10) = -10(x - (-2))$
$y + 10 = -10x - 20$
$y = -10x - 30$